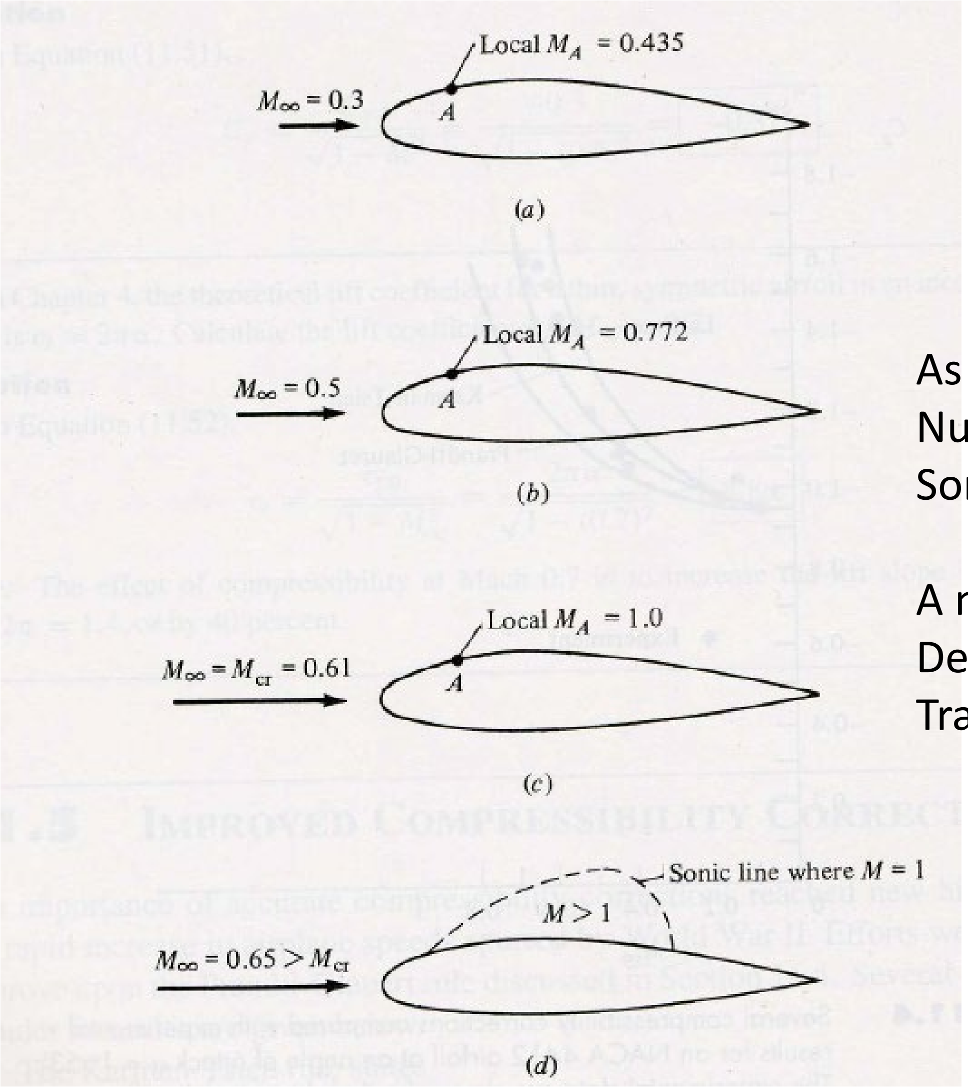
AE 3030: Lecture 13
Transonic Flow

L. Sankar
lsankar@ae.gatech.edu

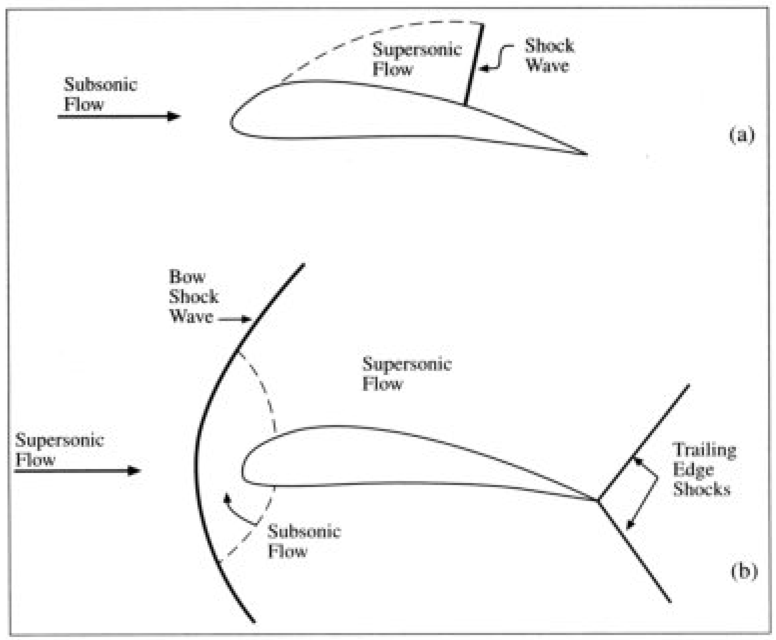
Variation of Flow Field with Increasing Freestream Mach Number



As freestream Mach
Number increases,
Sonic conditions are reached.

A mixed subsonic-supersonic flow
Develops. We call such mixed flow
Transonic flow.

In Supersonic Flow, the shock may move aft and become oblique
Drag Coefficient Decreases



http://www.dept.aoe.vt.edu/~mason/Mason_f/ConfigAeroTransonics.pdf

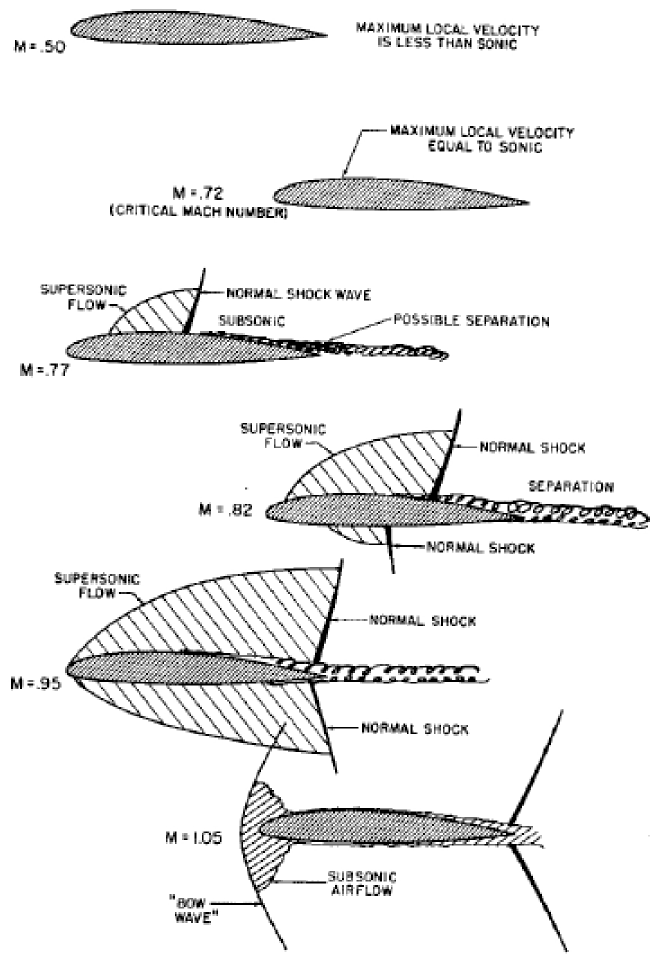


Figure 3.9. Transonic Flow Patterns (sheet 1 of 2)

Figure 7-1. Progression of shock waves with increasing Mach number, as shown in *Aerodynamics for Naval Aviators*,[2] a classic Navy training manual (not copyrighted).

Variation of Inviscid Pressure Distribution with Freestream Mach Mumber

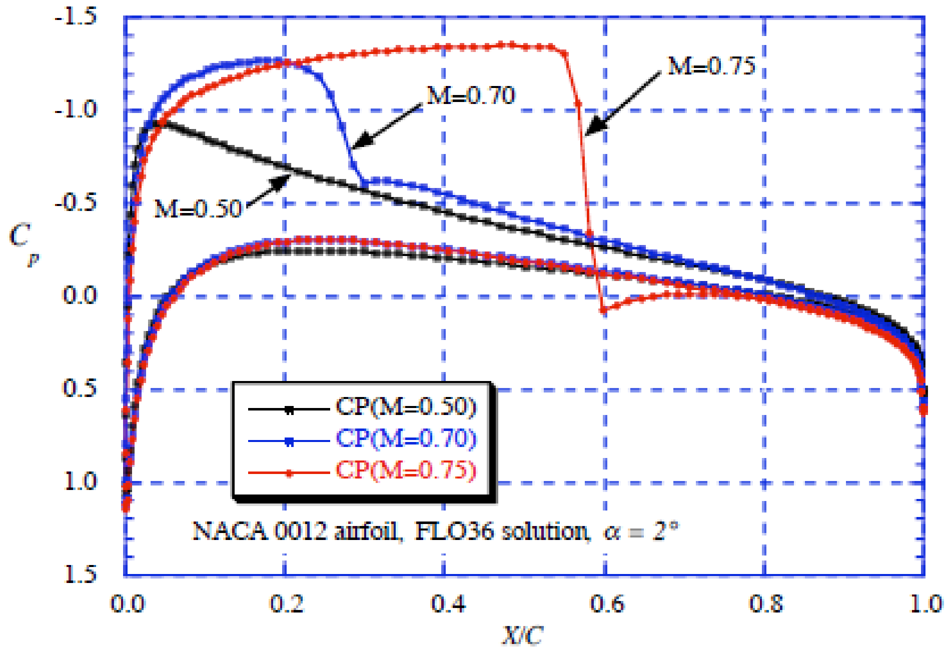
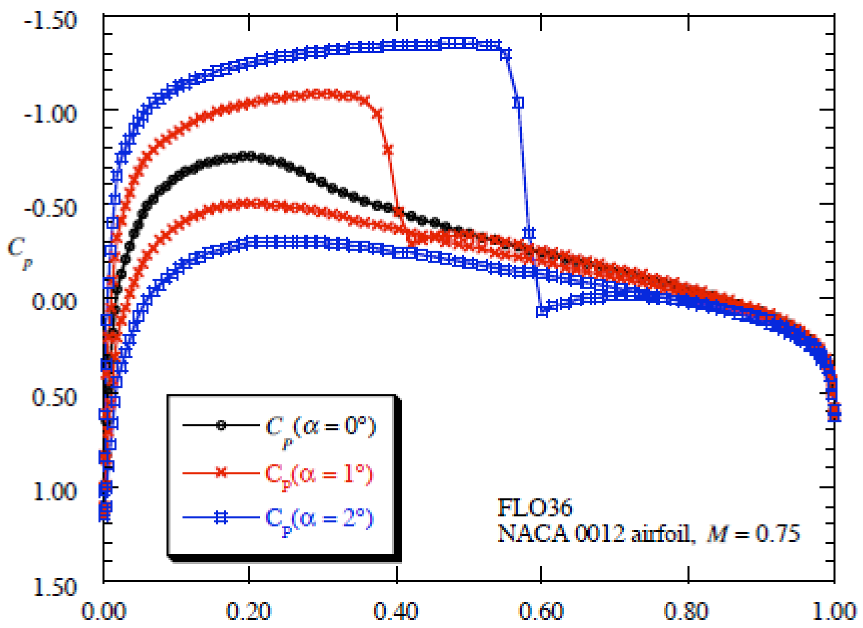
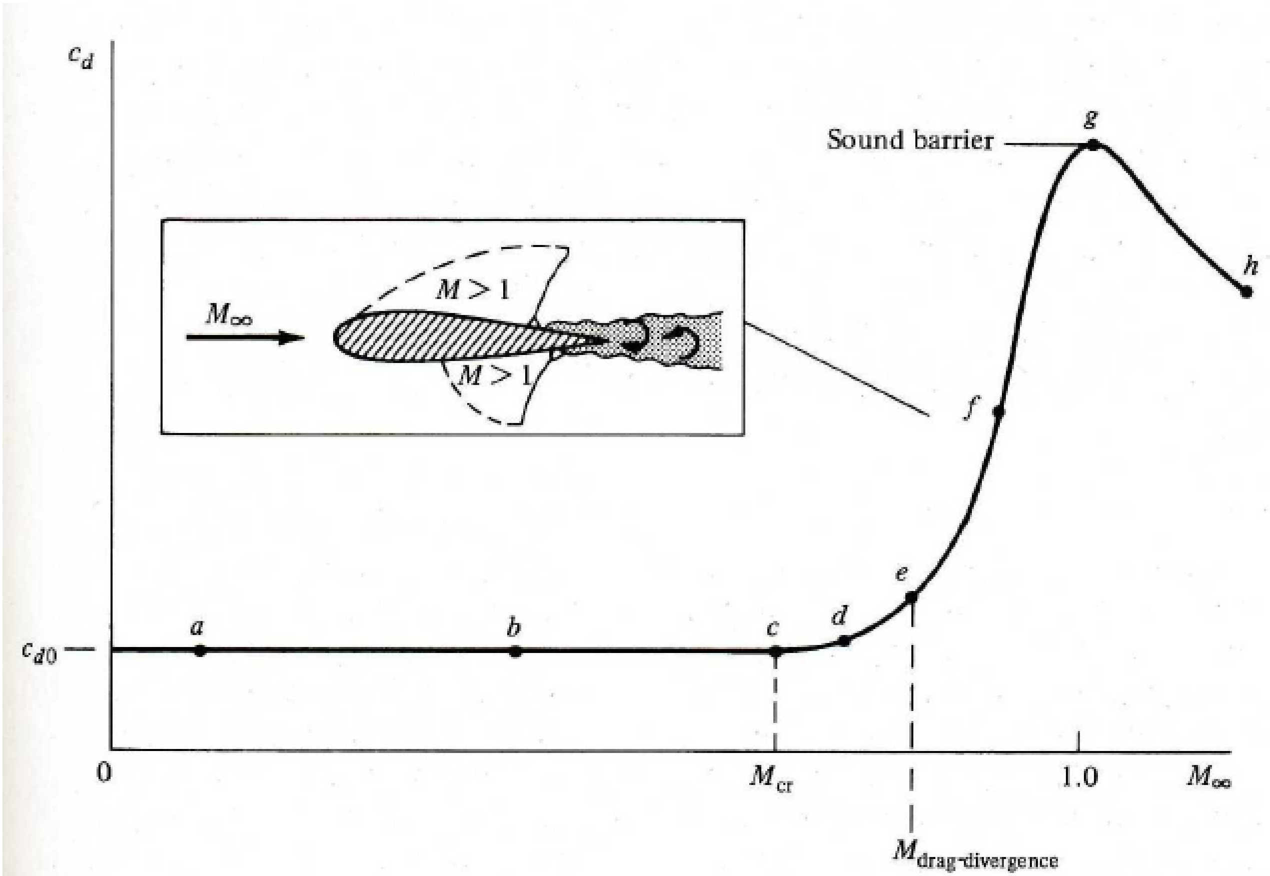


Figure 7-4. Pressure distribution change with increasing Mach number, NACA 0012 airfoil, $\alpha = 2°$.

Variation of C_p with Alpha



Drag divergence Mach Number



Deag Divergence Mach Number

- The Mach number at which the drag coefficient begins to rise rapidly due to transonic shock effects.
- A number of quantitative definitions exist. One common definition is: the Mach number at which dC_d/dM_∞ first exceeds 0.1.

$$\frac{\partial C_D}{\partial M} = 0.1$$

Lift also increases with freestream Mach Number

- Prandtl-Glauert rule states

$$C_p = \frac{C_{p,incompressible}}{\beta} = \frac{C_{p,incompressible}}{\sqrt{1 - M_\infty^2}}$$

$$C_{l,\,compressible} = \frac{1}{\beta} C_{l,\,incompressible}$$

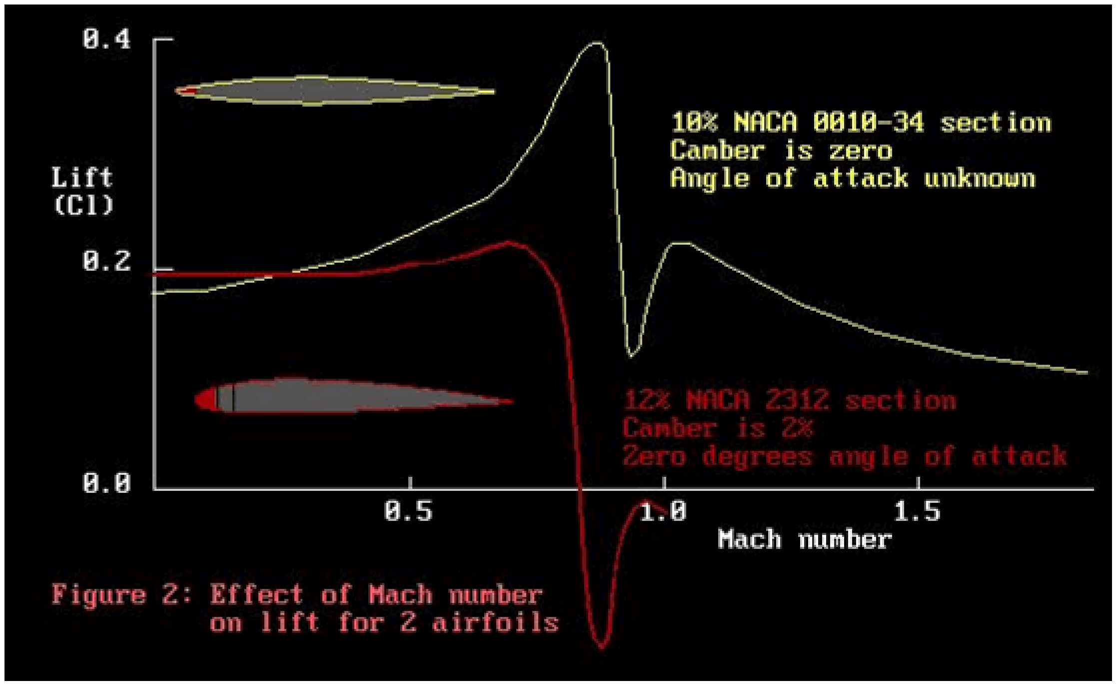
$$C_{d,\,compressible} = \frac{1}{\beta} C_{d,\,incompressible} = 0$$

$and,$

$$C_{m,\,compressible} = \frac{1}{\beta} C_{m,\,incompressible}$$

if the airfoil shapes are the same.

Variation of Lift with Mach Number



Effect of L/D on Range

- Breguet Range Equation

$$R = \frac{V}{sfc}\frac{L}{D}log(\frac{W_o + W_f}{W_o})$$

R: Range
V : Freestream Velocity
sfc: Specific fuel consumption, lbm fuel burned per hour per pound of thrust
W_0: Weight at the end of flight
W_f: Fuel Burnt during flight

In Transonic Regime, V times L/D can be very high, maximizing range.

Commercial aircraft like to operate in the transonic regime for this reason!

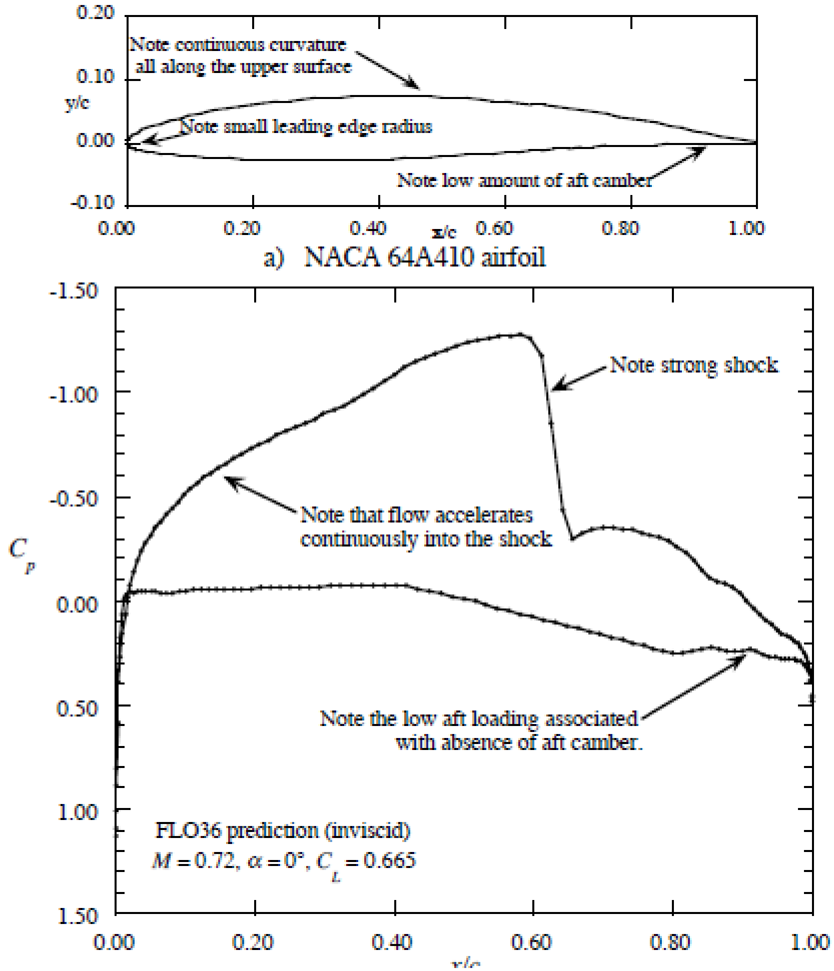
Improving Transonic Performance

- Supercritical Airfoil Technology
- Area Rule
- Sweep

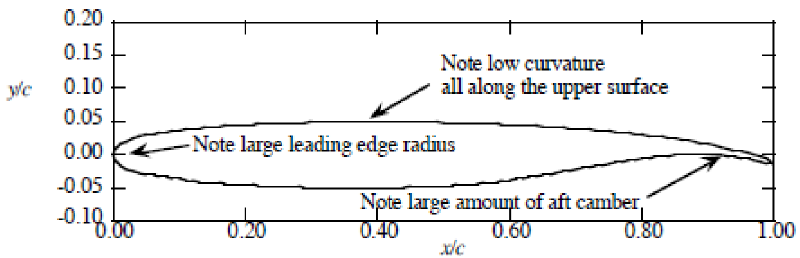
Supercritical Airfoil Technology

- Robert Whitcomb developed at NASA Langley airfoils that had weak or no shock waves.
  - He called them supercritical airfoils.
- During the early 1970s these were refined with test data.
- During 1970s, transonic potential flow methods became available for analyzing and designing supercritical airfoils.
- During 1980s, Euler and Navier-Stokes equations solvers (CFD) became available.

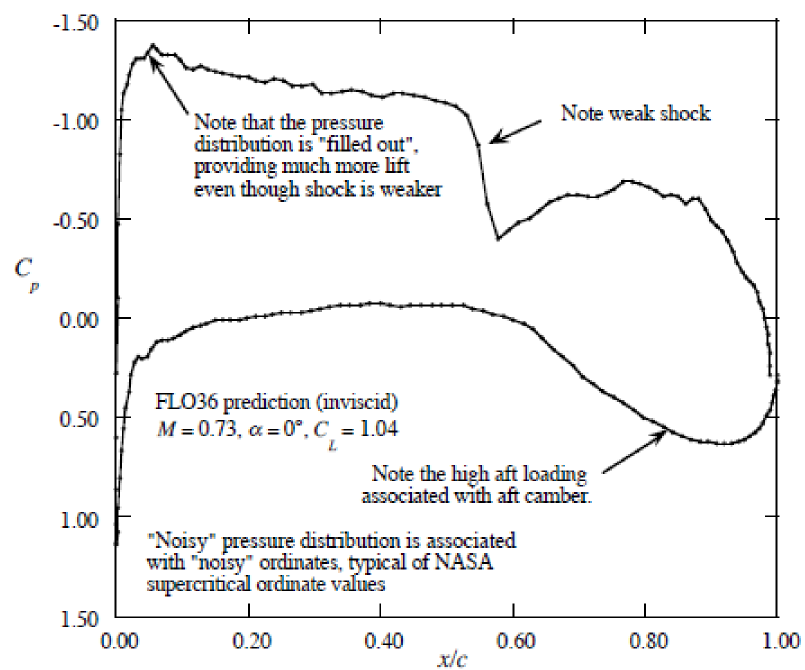
Pressure Distribution over Classical NACA Airfoils



Pressure Distribution over Supercritical Airfoils



a) FOIL 31



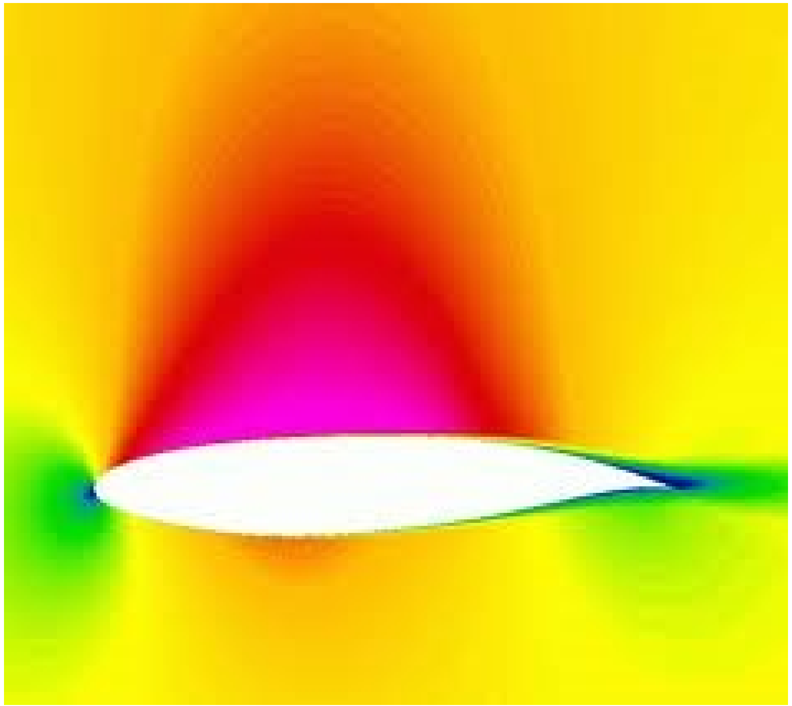
Blunt Leading Edge

Upper surface flattens once local Mach number reaches slightly above 1 to avoid further expansion.

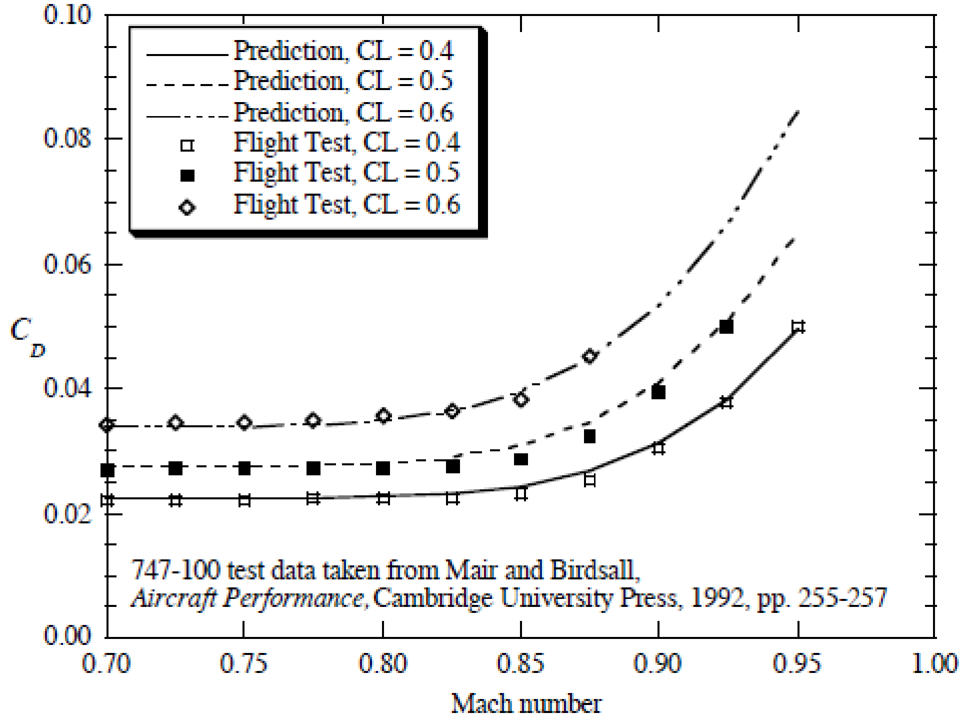
Lower surface shape is adjusted to make–up for loss in airfoil thickness to chord ratio

Cove region on the lower surface near the trailing edge gives some extra lift.

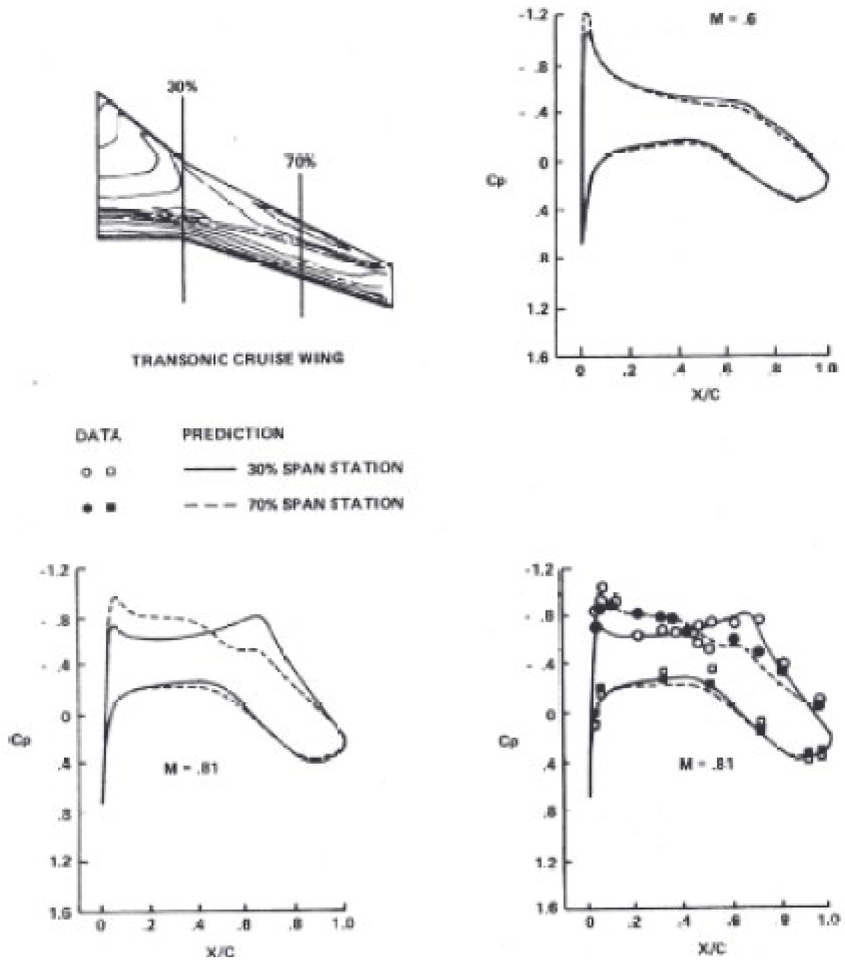
Very Weak Shocks..
Pressure Contours over a Supercritical Airfoil



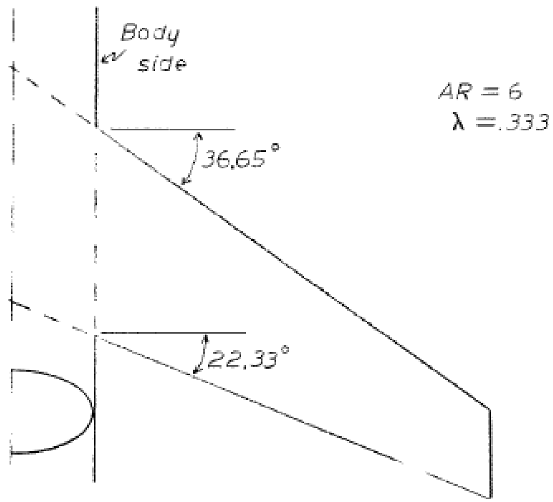
Transonic Wings Behave Similarly



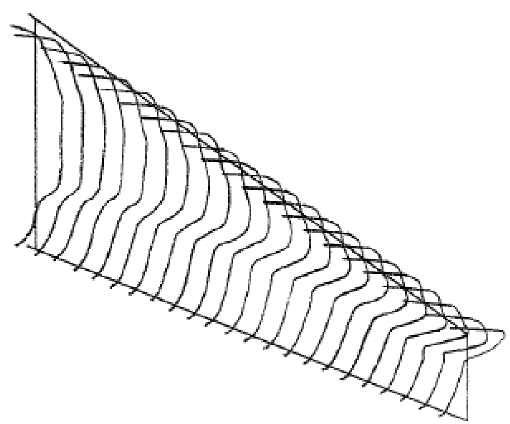
Transonic Flow over Supercritical Wings



Upper Surface Pressures for a Wing tested by Royal Aeronautical Establishment (RAE), England

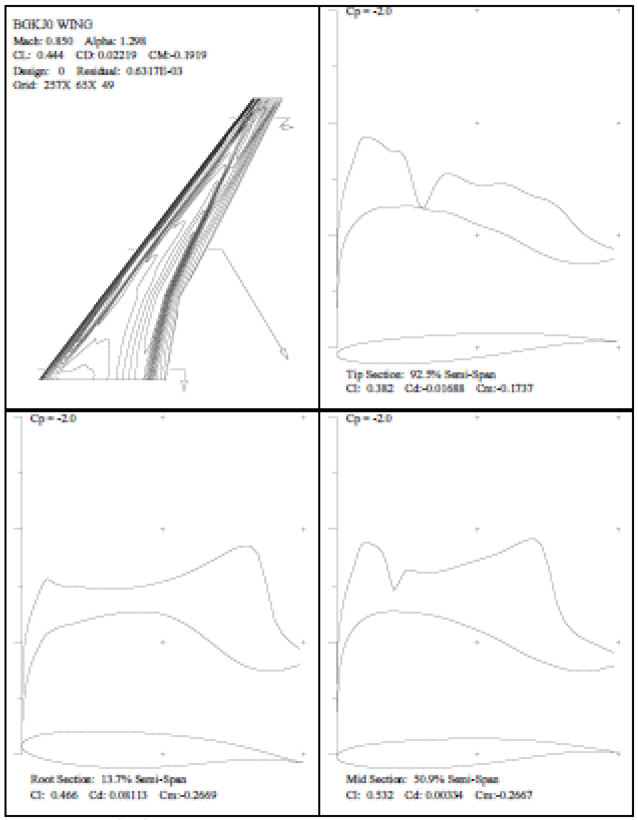


(a) RAE "A" configuration

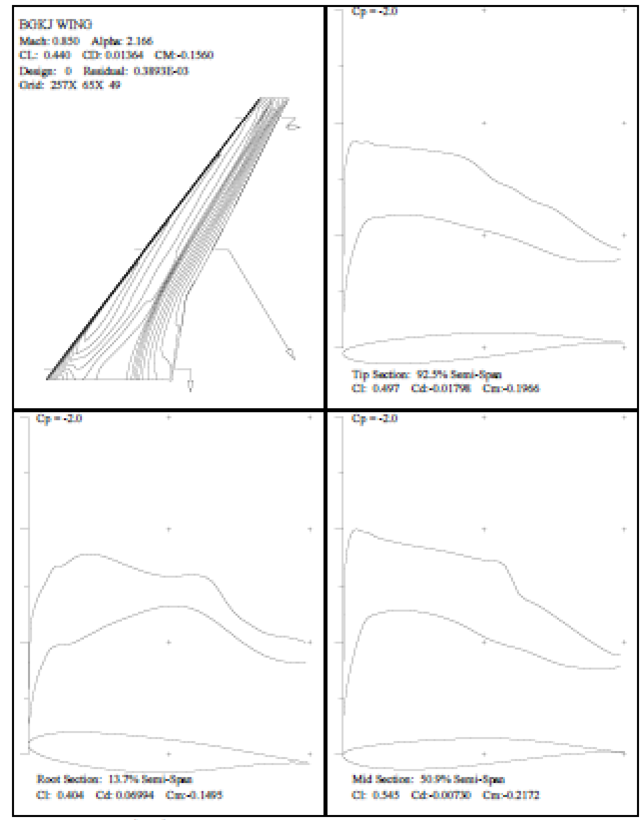


(b) Upper surface pressure distribution
FIG. 33 RAE "A"

Transonic Wing Design

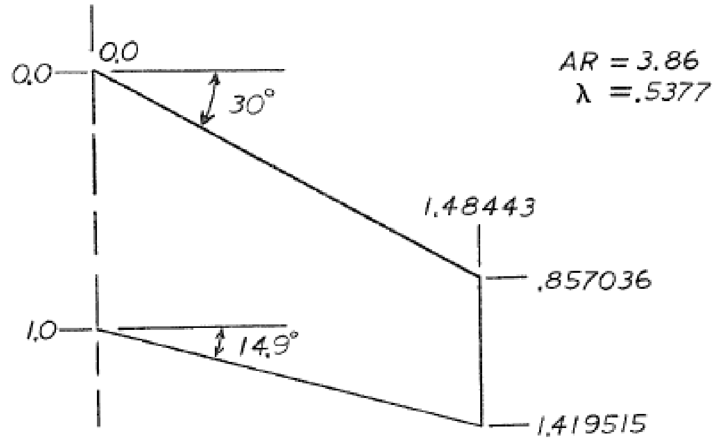


(a) Before Optimization

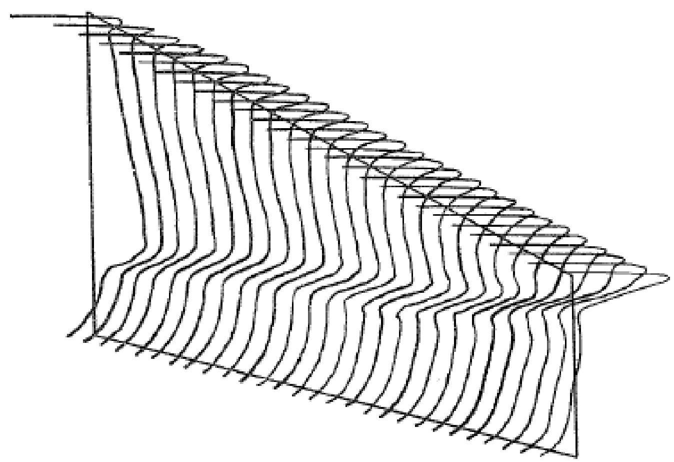


(b) After Optimization

Short Aspect ratio Wing tested at ONERA
ONERA M6 WING



(a) ONERA M6 configuration

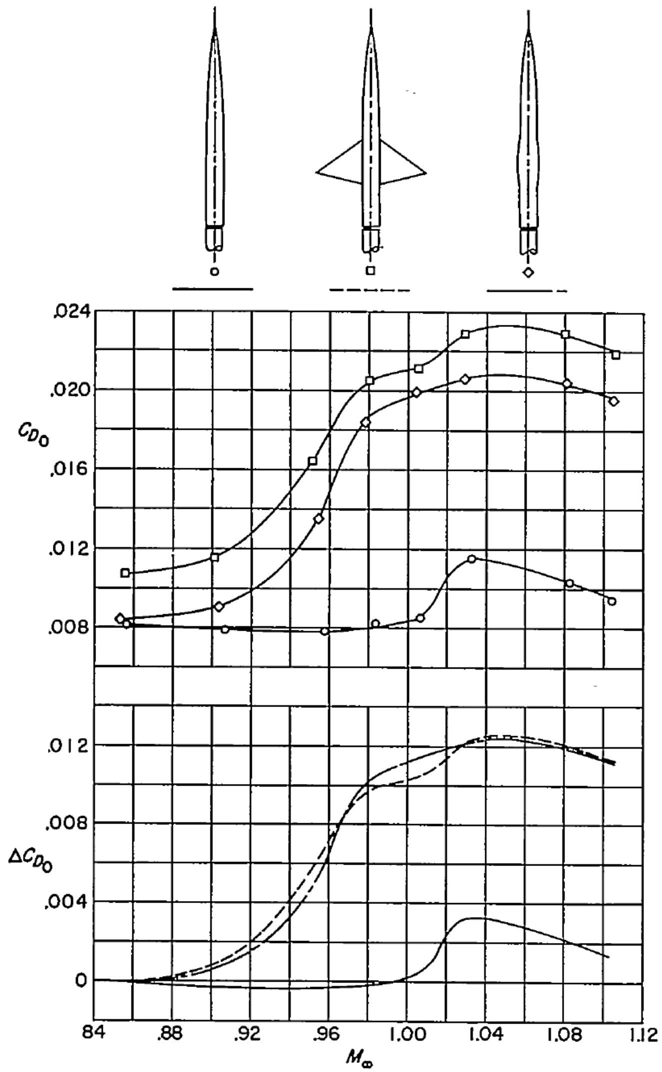


(b) Upper surface pressure distribution

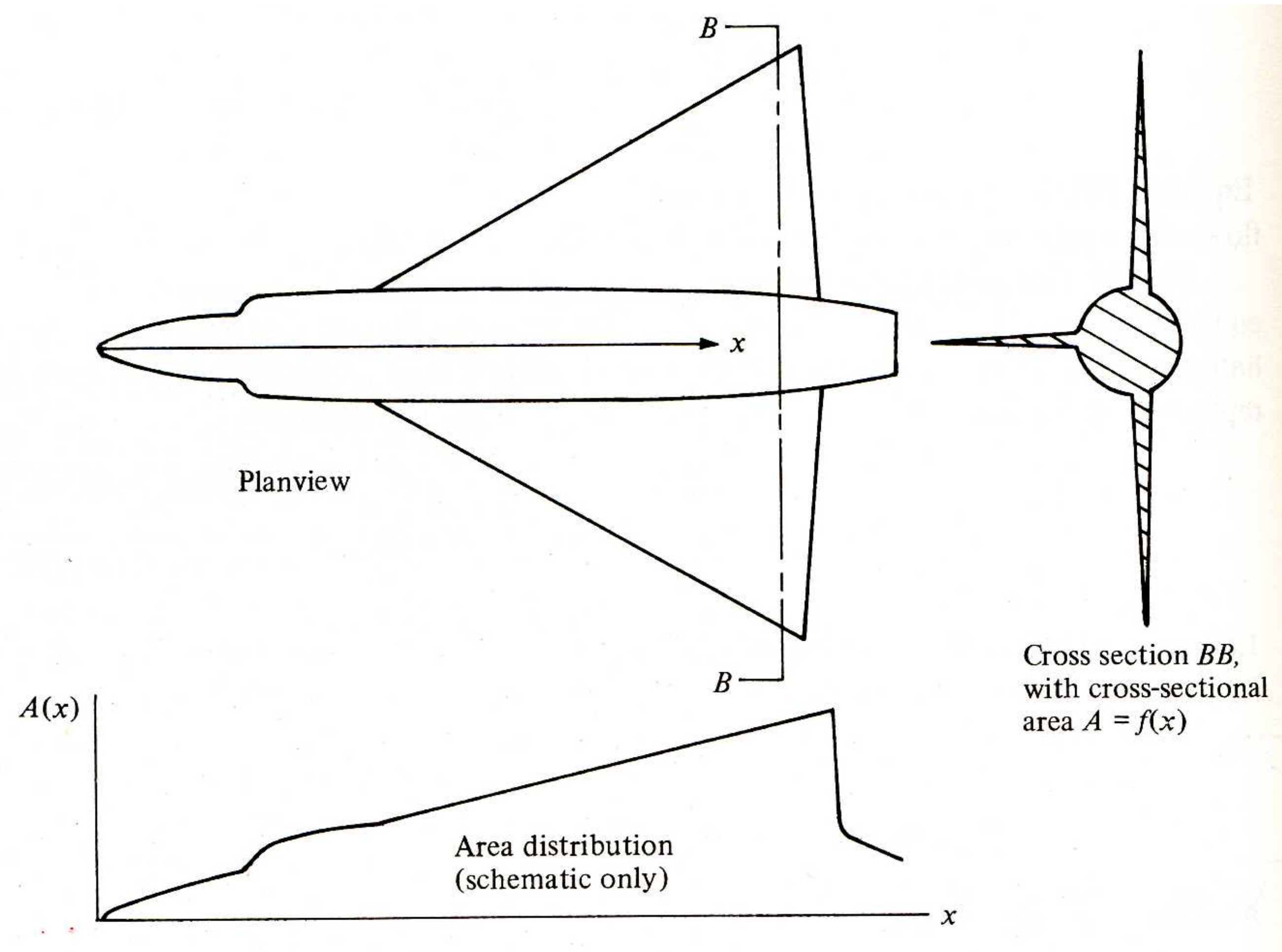
Whitcomb's Area Rule

- Described in the NACA Report 1273, available at the t-square site.
- Whitcomb tested many wing body configurations. (NACA RM L52D01, 1952).
- He found that the measured drag coefficient was comparable to that of bodies of revolution of equivalent cross section areas.
- He also found that abrupt changes in the cross section can increase transonic wave drag.

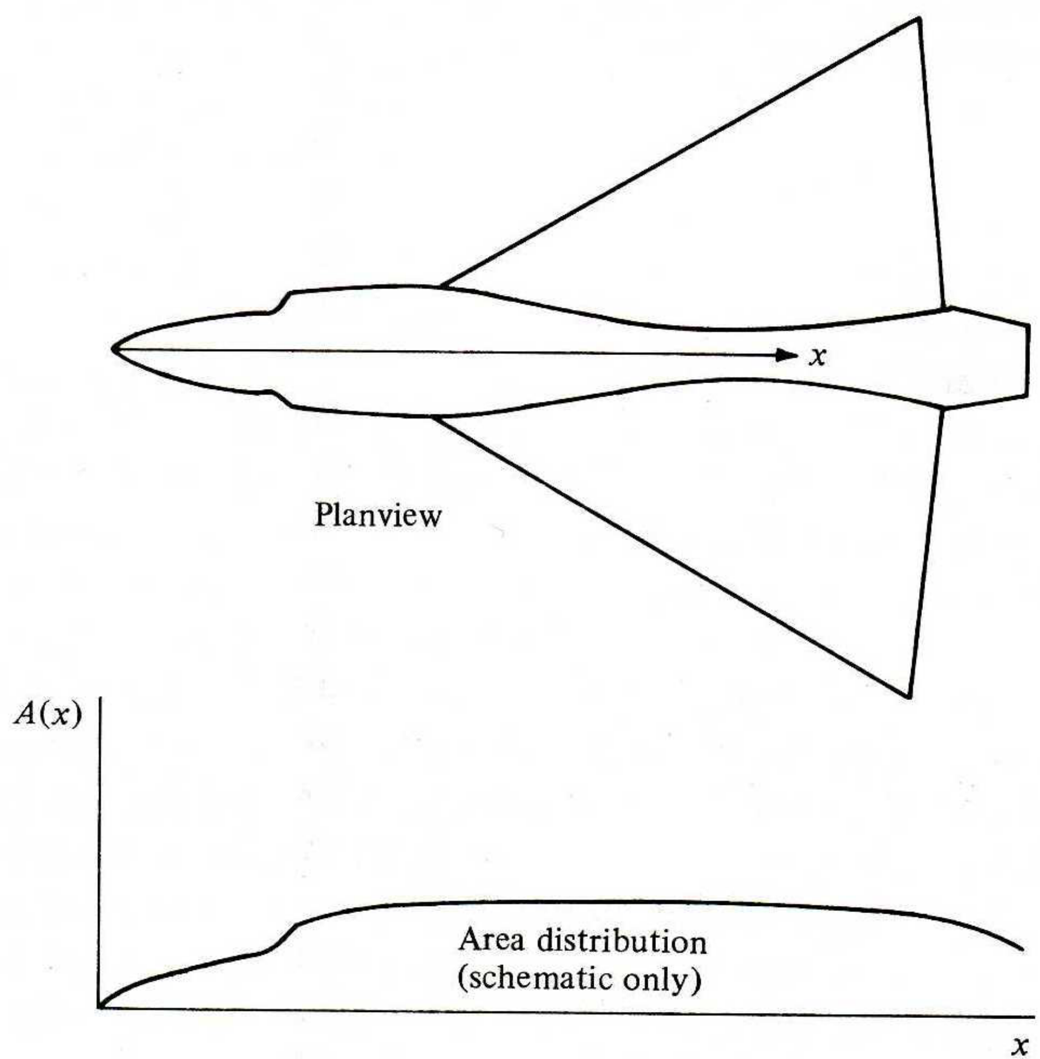
Whitcomb’s Observation



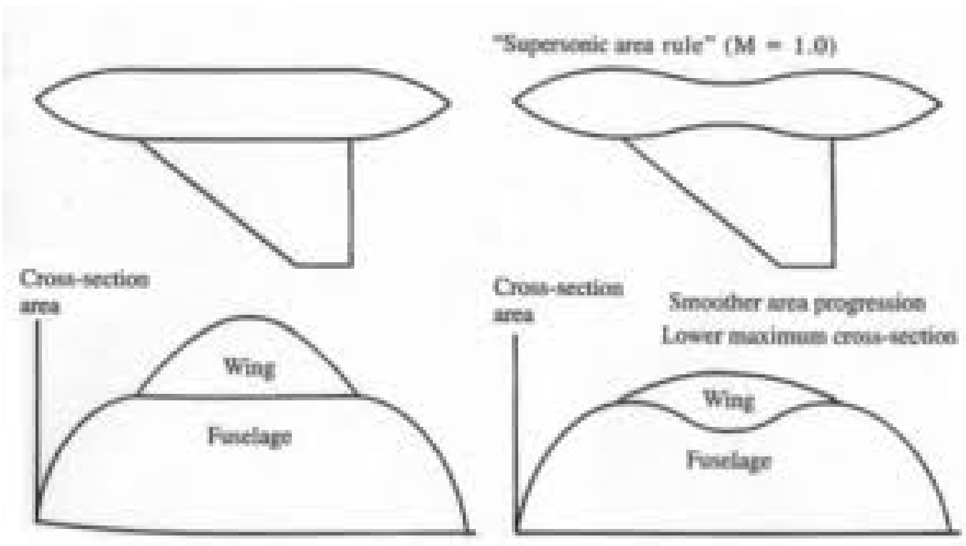
Drag Rise in Transonic Flow is associated with Rapid Changes to Cross Section



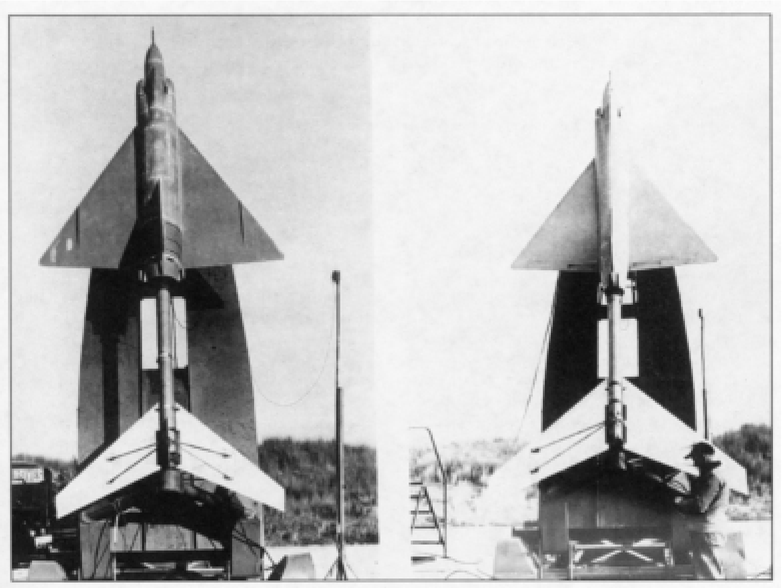
Area Rule



Cross-Sectional Area Varies Smoothly



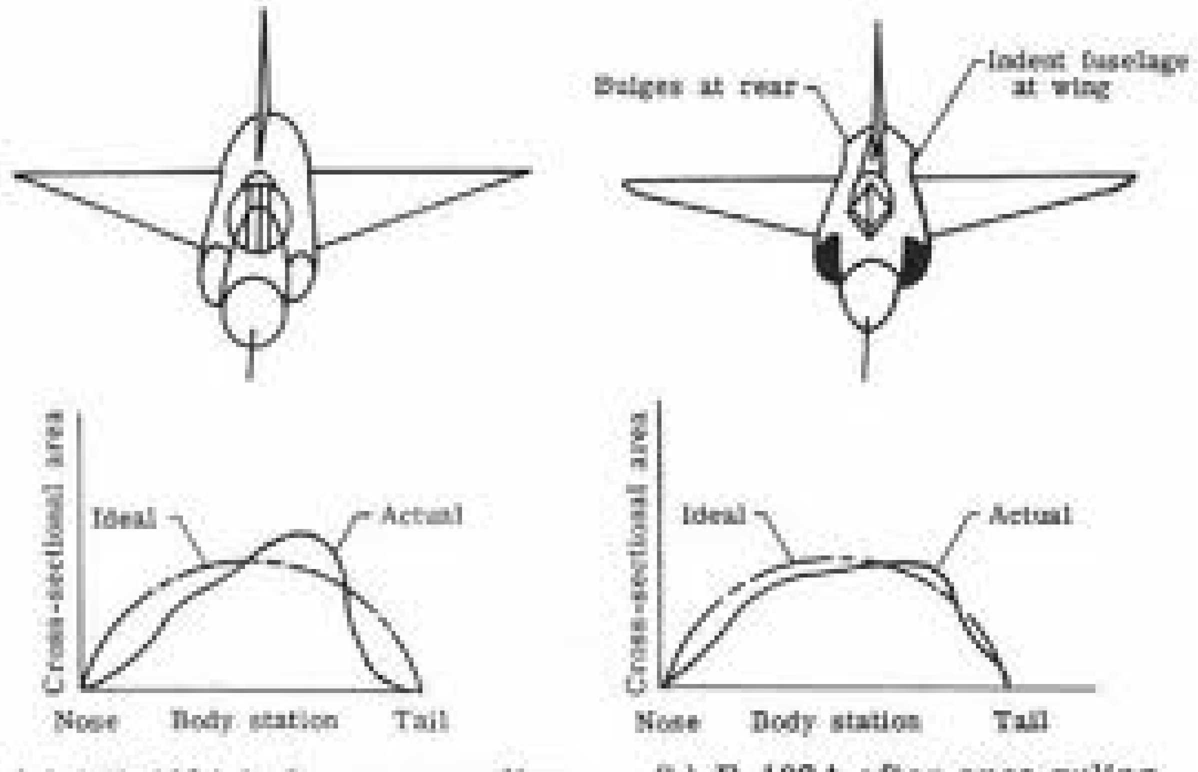
Convair F-102 Modified with Area Rule



http://history.nasa.gov/SP-4219/Chapter5.html

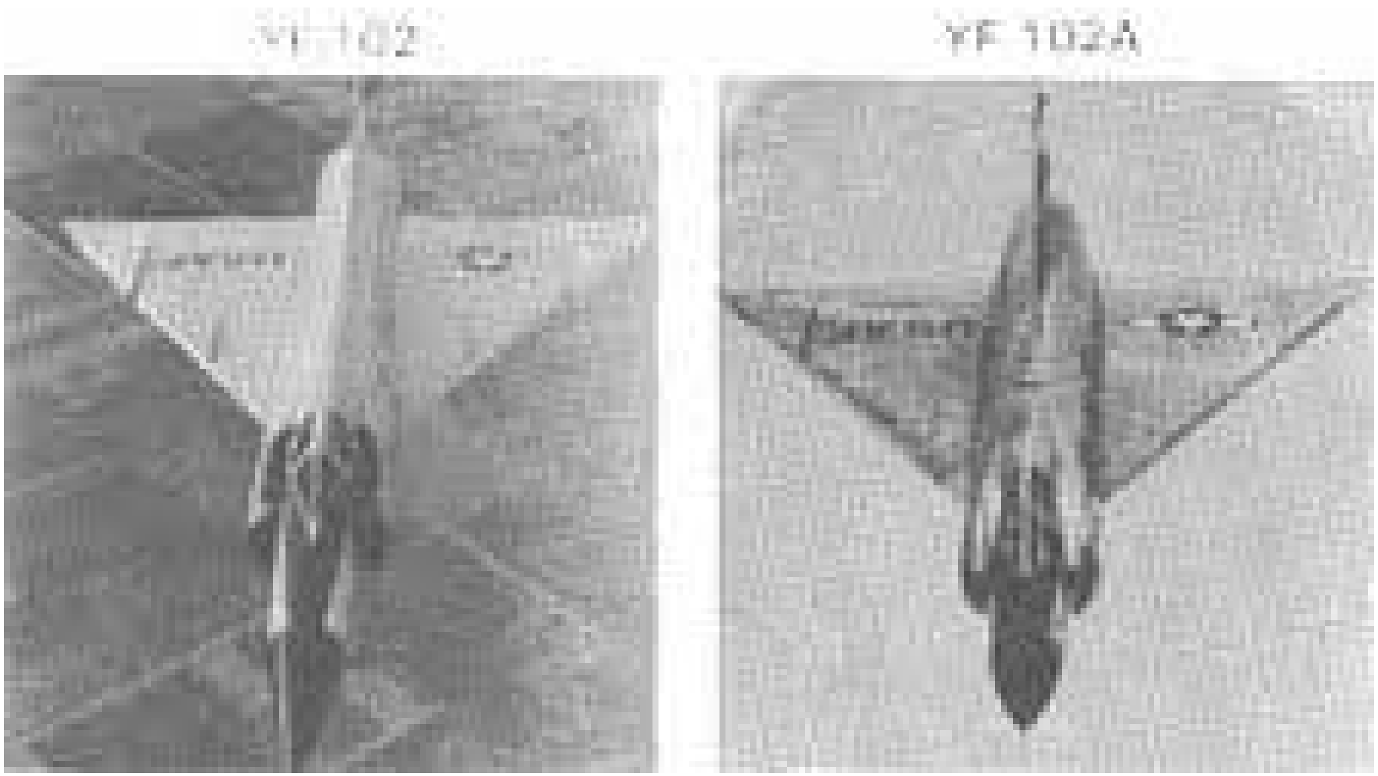
In mid-August 1952, a group of Convair engineers were at Langley to observe the performance of the F102 model in the Eight-Foot High-Speed Tunnel. Shown the disappointing test results, the engineers asked the Langley engineers if they had any suggestions. Whitcomb's first research memorandum on the area rule would not be published for another month, but he had completed his tests on the various wing-body combinations using indented fuselage shapes. He explained his findings and the area rule concept to the Convair team.

F-102 Before and After

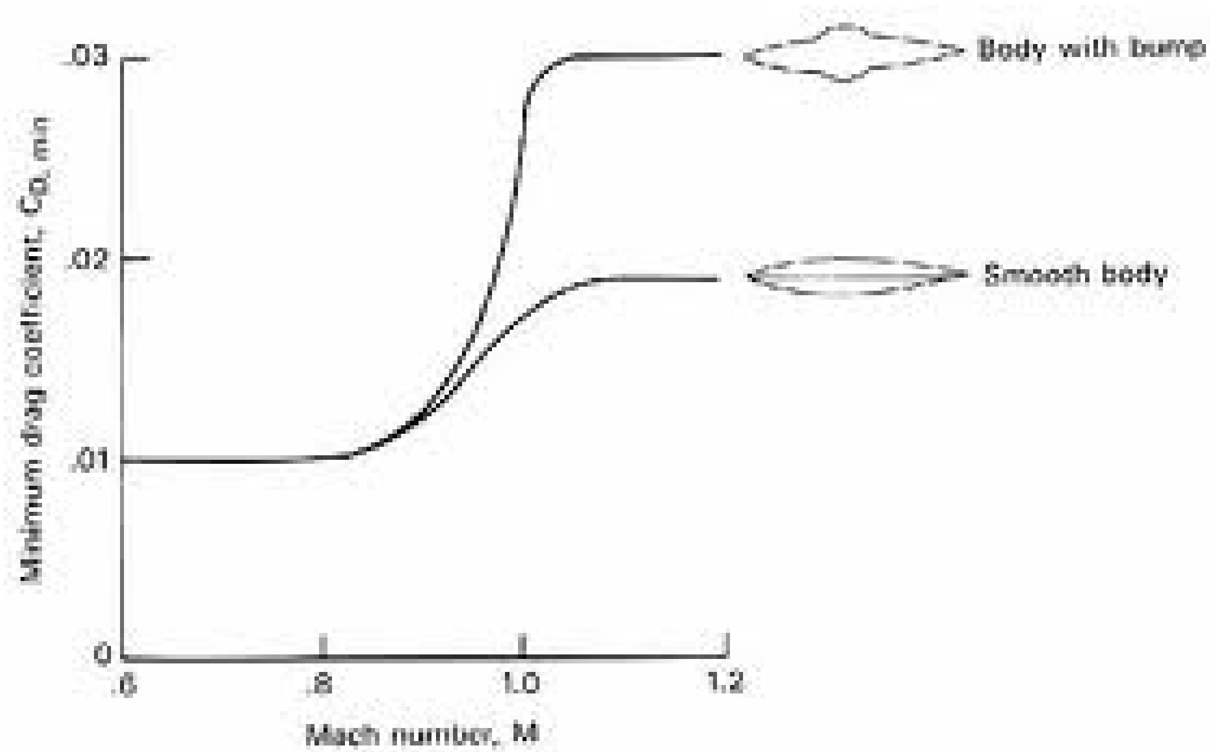


(a) YF-102A before area ruling.
(b) F-102A after area ruling.

F-102 Before and After



Effect of Area Rule on Drag



Other Examples

- The success of the area rule-based F-102 and F11F was followed by the incorporation of the area rule in virtually every supersonic aircraft built after that point.
- The Vought F8U "Crusader" fighter and the Convair B-58 "Hustler" bomber, both of which were on the drawing board at the time the area rule was developed, were redesigned using Whitcomb's approach.
- The F-106, which was Convair's follow-on design to the F-102A, adhered even more to the area rule. It was able to incorporate a much deeper indentation in the fuselage than its predecessor, because it was an entirely new aircraft, unencumbered by existing design elements.
- The fuselage of the Republic F-105 "Thunderchief" fighter/bomber, which flew for the first time in 1955, incorporated the area rule in a slightly different manner. It could not be indented because of its complex engine inlets, so a bulge was added to the aft region of the fuselage to reduce the severity of the change in the cross sectional area at the trailing edge of the wing.
- The Rockwell B-1 bomber and the Boeing 747 commercial airliner also used the addition of a cross-sectional area to reduce their drag at transonic speeds.
- Both the B-1 and the 747 have a vertical "bump" in the forward section of the fuselage ahead of the wing. It is perhaps more visible in the 747, where it houses the airliner's characteristic second story, but both airframe modifications were added to smooth the curve of the design's crosssectional area

Sources http://history.nasa.gov/SP-4219/Chapter5.html#Chapt5-41

41. Whitcomb, interview, May 2, 1995; Whitcomb, "Research on Methods for Reducing the Aerodynamic Drag at Transonic Speeds," November 14, 1994, p. 3.

42. Bill Robie, *For the Greatest Achievement: A History of the Aero Club of America and the National Aeronautic Association*, (Washington, DC: Smithsonian Institution Press, 1993), p. 232; Richard T. Whitcomb, telephone interview with author, May 15, 1995.

NACA/NASA Langley engineer Richard T. Whitcomb was awarded the 1954 Collier Trophy for his development of the "area rule, " an innovation that revolutionized the design of virtually every transonic and supersonic aircraft ever built. Here Whitcomb inspects a research model in the 8-Foot Transonic Tunnel at Langley. (NASA photo no. LAL 89118).

Effect of Area Rule on Drag Rise

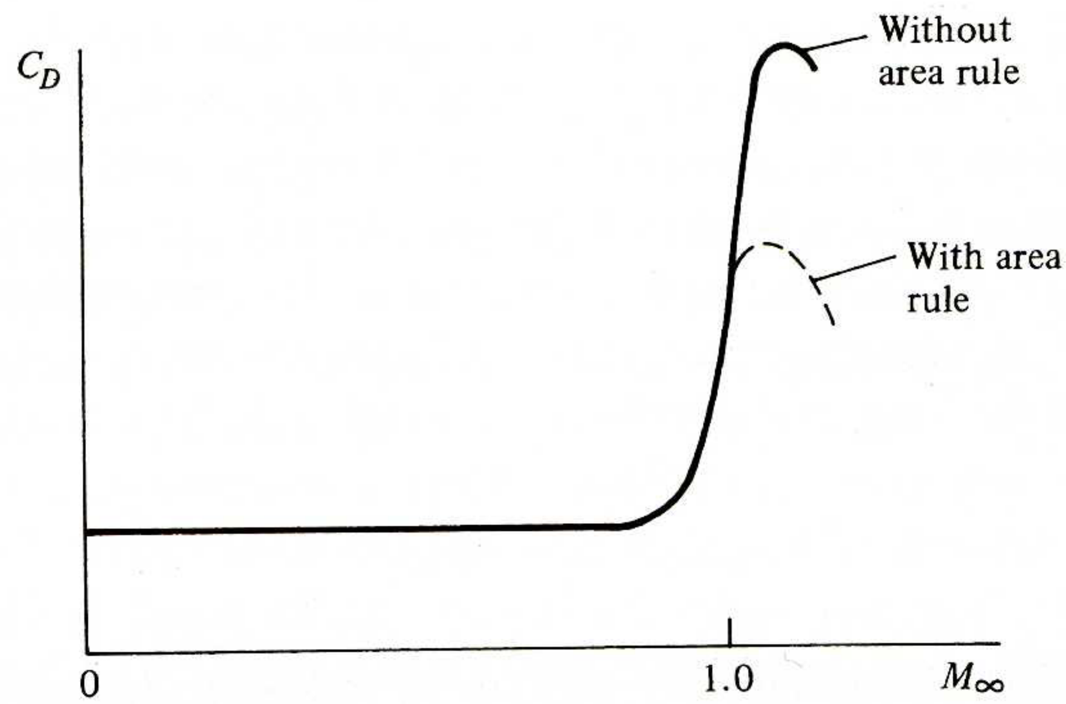


Figure 11.18 The drag-rise properties of area-ruled and non-area-ruled aircraft (schematic only).

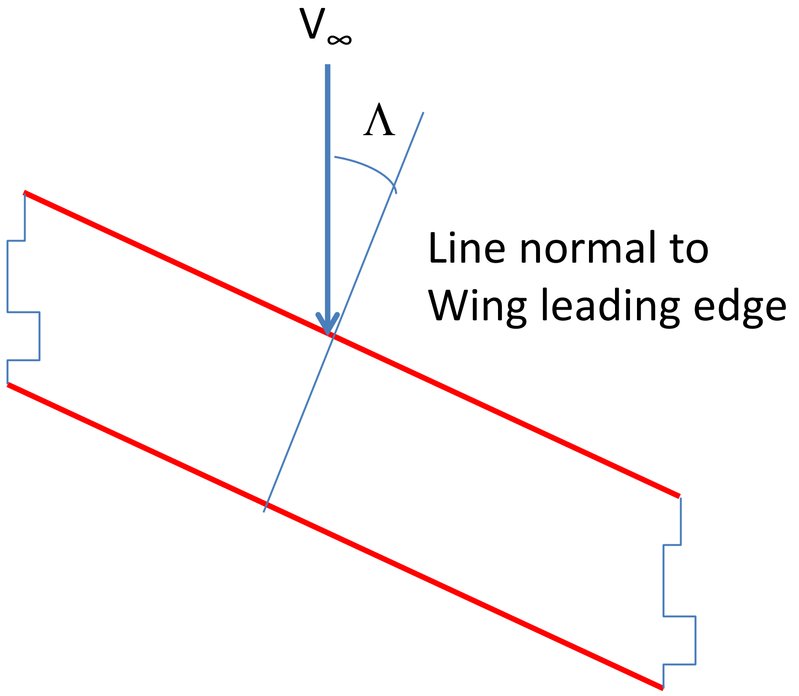
Effects of Wing Sweep on Subsonic Aerodynamics

Swept Wing: Historical Development http://en.wikipedia.org/wiki/Swept_wing

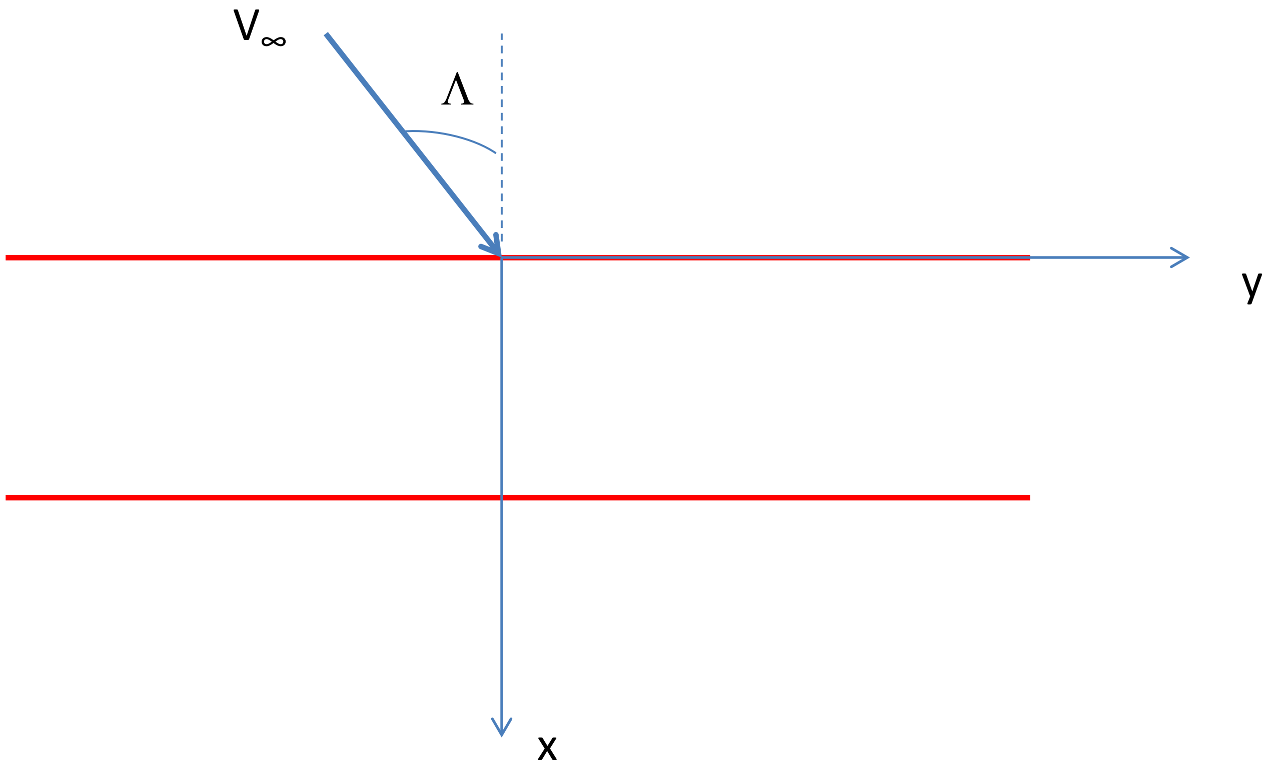
- Introduced in Germany around 1935, to improve transonic performance- reduce wave drag.
  - Dr. Adolf Busemann proposed it, tests were done at Göttingen, knowledge passed onto Messerschmitt.
  - Jet engines with increased thrust and low drag allowed aircraft to achieve transonic and supersonic speeds.
- Technology evolved thru' end of WW II.
- In 1945, Robert T. Jones at NACA pointed out the benefits of swept wings.
  - Theoreticians called it hocus pocus, and demanded real mathematics.
- Boeing built B-47 Stratojet with swept wing around 1951.
- Korean War (1950-1953) led to Mig -15 and F-86 SabreJet.
  - F-86 was influenced by Busemann's work.

Theory

- We are still working with 2-D aerodynamics.
- We therefore consider swept wings of infinite aspect ratio.
- The sweep angle is given the symbol Λ.



We rotate the picture..
Define x and y- axes



Governing Equation

- In an earlier lecture, we derived the 2-D non-linear potential flow equation:

$$\left(a^2 - u^2\right)\frac{\partial^2\phi}{\partial x^2} - (2\mathrm{uv})\frac{\partial^2\phi}{\partial x\partial y} + \left(a^2 - \mathrm{v}^2\right)\frac{\partial^2\phi}{\partial y^2} = 0$$

The 3-D form of this non-linear potential flow equation is given by

$$\left(a^2 - u^2\right)\frac{\partial^2\phi}{\partial x^2} - (2\mathrm{uv})\frac{\partial^2\phi}{\partial x\partial y} + \left(a^2 - \mathrm{v}^2\right)\frac{\partial^2\phi}{\partial y^2}$$

$$+ \left(a^2 - w^2\right)\frac{\partial^2\phi}{\partial z^2} - (2\mathrm{uw})\frac{\partial^2\phi}{\partial x\partial z} - (2\mathrm{vw})\frac{\partial^2\phi}{\partial z\partial y} = 0$$

Infinite Aspect Ratio Assumption

- Before we linearize the 3-D equation, we want to simplify it by getting rid of unneeded terms.
- The wing has an infinite span.
- Thus, there can be no spanwise (y-) derivatives of physical properties such as velocity.
- Even a small gradient of velocity in the spanwise direction will build up to infinitely large values along the span.

Infinite Aspect ratio Assumption

- We thus require:

$$\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial \mathrm{v}}{\partial y} = 0 \qquad \frac{\partial \mathrm{w}}{\partial y} = 0$$

Use the definition of velocity in terms of velocity potential:

$$\vec{V} = \nabla \phi$$

$$u = \frac{\partial \phi}{\partial x} \qquad \mathrm{v} = \frac{\partial \phi}{\partial y} \qquad \mathrm{w} = \frac{\partial \phi}{\partial z}$$

We get

$$\frac{\partial^2 \phi}{\partial x \partial y} = 0 \qquad \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \frac{\partial^2 \phi}{\partial z \partial y} = 0$$

Our non-linear potential equation simplifies

The original equation

$$\left(a^2-u^2\right)\frac{\partial^2\phi}{\partial x^2}-(2\mathrm{uv})\frac{\partial^2\phi}{\partial x\partial y}+\left(a^2-\mathrm{v}^2\right)\frac{\partial^2\phi}{\partial y^2}$$
$$+\left(a^2-w^2\right)\frac{\partial^2\phi}{\partial z^2}-(2\mathrm{uw})\frac{\partial^2\phi}{\partial x\partial z}-(2\mathrm{vw})\frac{\partial^2\phi}{\partial z\partial y}=0$$

Takes on the shorter form, very similar to our 2-D form:

$$\left(a^2-u^2\right)\frac{\partial^2\phi}{\partial x^2}+\left(a^2-\mathrm{w}^2\right)\frac{\partial^2\phi}{\partial z^2}-(2\mathrm{uw})\frac{\partial^2\phi}{\partial x\partial z}=0$$

This form is very similar to our 2-D non-linear form shown earlier!

WE can linearize this equation for subsonic and supersonic flow the same way we did in 2-D

Linearization for thin airfoil sections, mild camber, and small alpha

In earlier lectures (see lecture #4)

$$\left(a^2 - u^2\right)\frac{\partial^2 \phi}{\partial x^2} - (2\mathrm{uv})\frac{\partial^2 \phi}{\partial x \partial y} + \left(a^2 - \mathrm{v}^2\right)\frac{\partial^2 \phi}{\partial y^2} = 0$$

was linearized in subsonic and supersonic flows, avoiding transonic flow regions Where linearization is not possible, as follows:

$$\left(a_\infty^2 - u_\infty^2\right)\frac{\partial^2 \phi}{\partial x^2} + a_\infty^2 \frac{\partial^2 \phi}{\partial y^2} = 0$$

By a similar process, our equation for Infinite aspect ratio swept wing

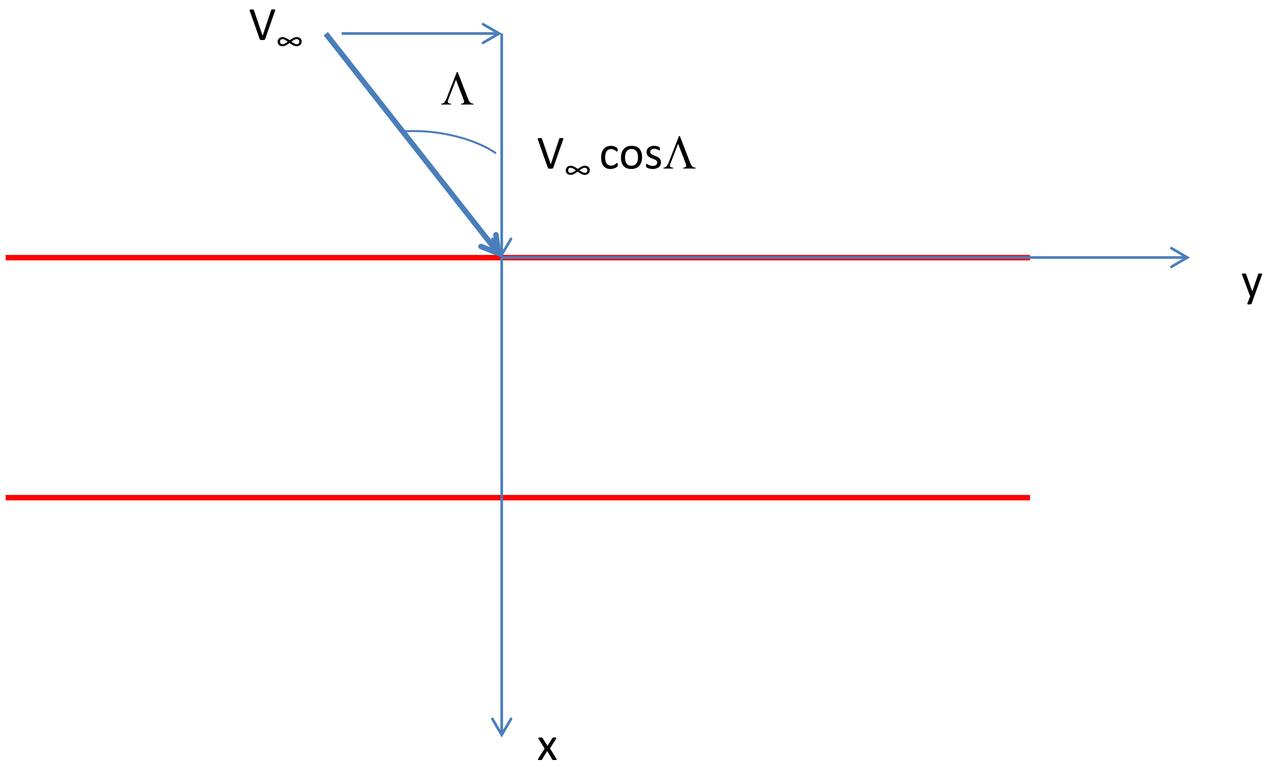
$$\left(a^2 - u^2\right)\frac{\partial^2 \phi}{\partial x^2} + \left(a^2 - \mathrm{w}^2\right)\frac{\partial^2 \phi}{\partial z^2} - (2\mathrm{uw})\frac{\partial^2 \phi}{\partial x \partial z} = 0$$

linearizes to

$$\left(a_\infty^2 - u_\infty^2\right)\frac{\partial^2 \phi}{\partial x^2} + a_\infty^2 \frac{\partial^2 \phi}{\partial y^2} = 0$$

Here u_∞ is the component of freestream along The x-axis, given by $V_\infty \cos\Lambda$

Freestream Component along x-axis



Non-Dimensionalization of Governing Equation

$$\left(a_\infty^2 - u_\infty^2\right)\frac{\partial^2\phi}{\partial x^2} + a_\infty^2\frac{\partial^2\phi}{\partial y^2} = 0$$

Use the knowledge u_∞ is the component of freestream along the x-axis, given by $V_\infty \cos\Lambda$

Also use the definition of Mach number: $$M_\infty = \frac{V_\infty}{a_\infty}$$

We get: $$\left(1 - M_n^2\right)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$ where $M_n = M_\infty \cos\Lambda$

M_n is the Mach number normal to the wing leading edge. For swept wings, it will be lower than freestream Mach number by the factor $\cos\Lambda$.
This lowering of Mach number reduces compressibility effects, delaying shocks.

Summary

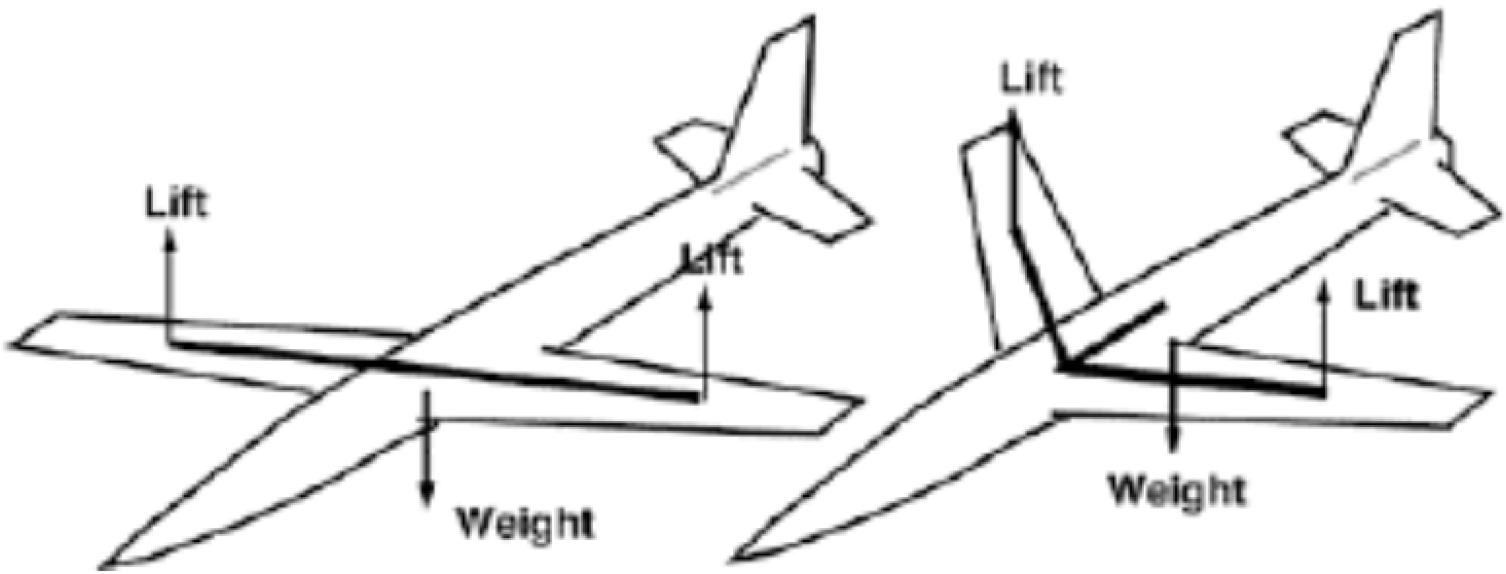
- Use a coordinate system where x axis is normal to leading edge.
- Analyze or use test data for this airfoil in this coordinate system.
- Compute the Mach number normal to the leading edge as $M_\infty \cos\Lambda$.
- Use this Mach number while computing Cp using P-G Rule, laitone’s Rule, or Karman-Tsien Rule.
- If this Mach number normal to leading edge is lower than critical Mach number, then transonic flow and shocks will not occur.

R.T. Jones Oblique Wing Concept
Wing had variable sweep

Flight demonstration of Oblique Wing Concept by NASA Dryden

Benefit of Oblique Wing

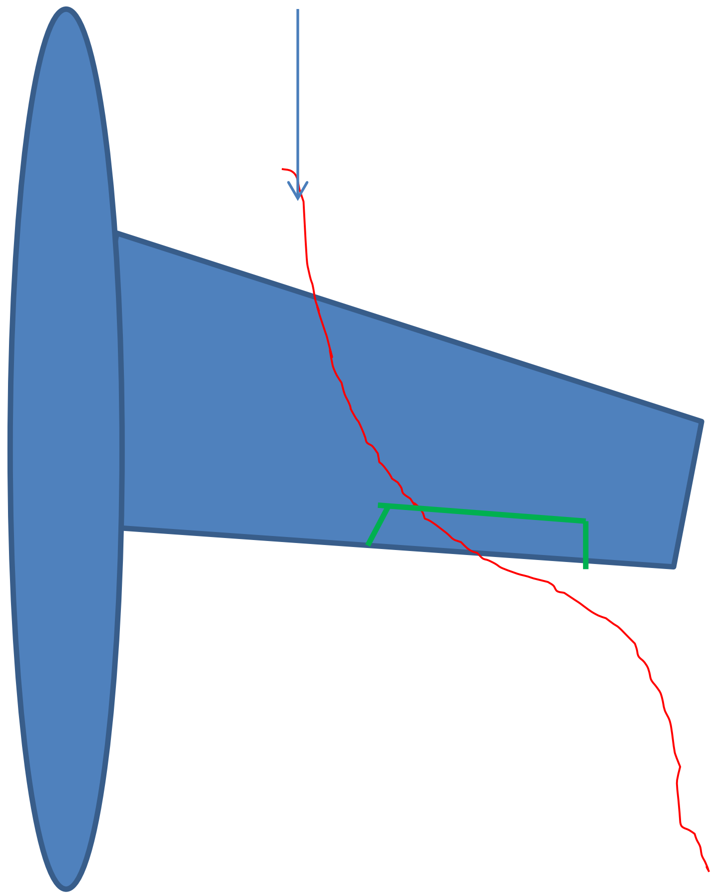
- Postpones shocks
- Structurally simple.
- Aerodynamic Center is always at a fixed place – at the hinge.



Benefits and Drawbacks of Swept Back Wing

- Benefits
  - Mach number normal to the leading edge is reduced.
  - Compressibility effects are reduced, shocks postponed or avoided.
- Drawbacks
  - Boundary layer may flow along the span (instead of leading edge to trailing edge), get progressively thicker.
  - Fluid particles in the boundary layer lose momentum more as they travel longer distances
  - This promotes tip stall, wing stall, and makes ailerons less effective.

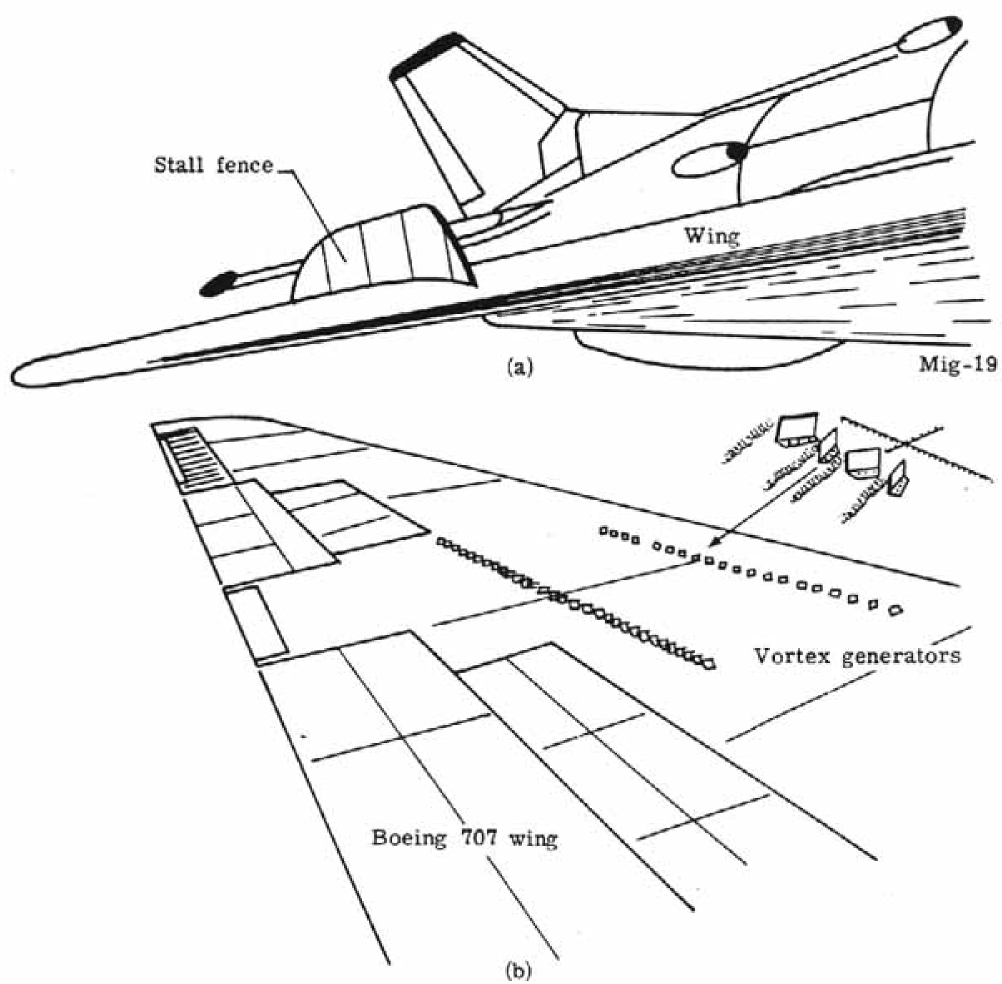
Spanwise Boundary Layer Flow



Boundary layer fences
are sometimes used to
obstruct spanwise flow
And keep wing fro stalling

http://en.wikipedia.org/wiki/File:Su-20_RB4.jpg

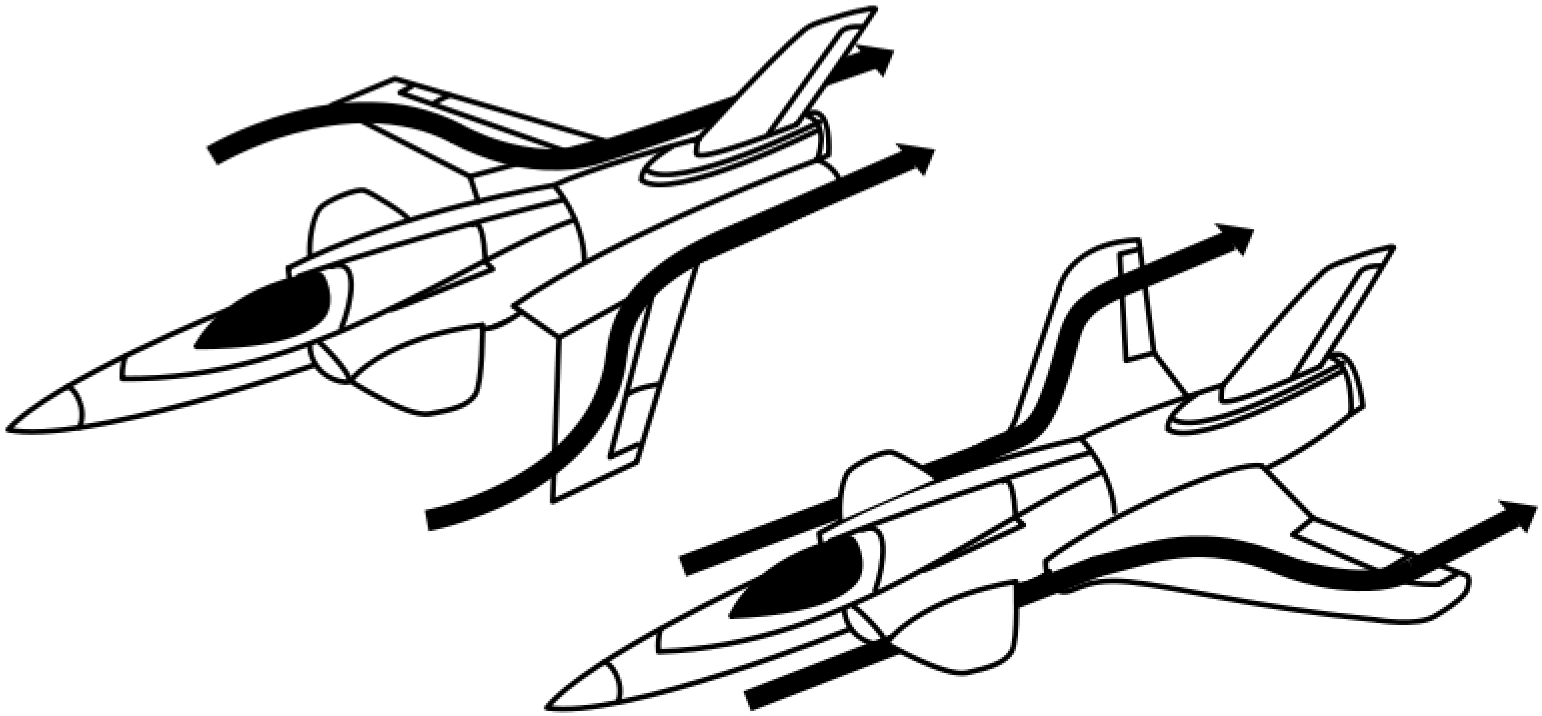
Stall fence vs Vortex Generator



Swept Forward Wings

- Swept forward wings have same benefits – postponement of transonic effects
- The boundary layer will have a tendency to flow from tip towards the root, causing root stall.
- Swept forward wing sections tend to twist so that the blade sections pitch up as the lift is increased.
- This pitch up can increase the lift further.
- Aeroelastic divergence (a progressive increase in twist and lift) until the wing structurally fails may occur.
- Swept-back wings do not have this difficulty.

Spanwise flow is the reverse of that fr the sweptback wing



Tip stall is avoided. Fuselage acts as a boundary layer fence reducing root stall.

Thus swept forward wings have greater high angle of attack performance, and greater maneuverability.
Aeroelastic divergence may be avoided with composite technology..

Summary

- We have completed Chapter 11.
  - We derived non-linear potential flow equation
  - We linearized the equation to come up with a equation similar to Laplace’s equation.
  - This allowed similarity rules (Prandtl-Glauert, Laitone, Karman-Tsien) that allow incompressible analyses or test data to be extended to compressible flow.
  - We studied critical Mach number, drag divergence Mach number, and transonic flow.
  - We studied how to use supercritical airfoils, area ruling, and wing sweep to postpone adverse effects of shock waves.
- All of the analyses were for 2-D airfoils or infinite aspect ratio swept wings.
- In the next lecture, we will look at finite wing theory.