Prescribed Wake Models for Rotors in Forward Flight

1. Rigid Wake:

In the rigid wake model, The tip vortex position is described by its age ϕ, which is the current blade azimuthal angle position minus the azimuthal angle of the shedding initiation point. The wake geometry is described in the inertial coordinate system as:

$$x = r\cos(\psi - \phi) + \mu_x \phi + \lambda_x \phi$$
$$y = r\sin(\psi - \phi) + \mu_y \phi + \lambda_y \phi$$
$$z = z_0 + \mu_z \phi - \lambda_z \phi \tag{1}$$

where the ψ is the azimuthal angle of the reference blade, and f is the vortex wake age.

The Glauert uniform inflow model is used to estimate the inflow components λ_x, λ_y and λ_z. The induced velocity in dimensionless form is:

$$\lambda = \mu \tan\alpha + \lambda_i = \mu \tan\alpha + \frac{c_T}{2\sqrt{\mu^2 + \lambda^2}} \tag{2}$$

<u>Landgrebe's Prescribed Wake Model:</u>

In this model, the x and y coordinates of the tip vortex are prescribed from a rigid wake model, as shown above. The vertical displacements of the tip vortices are given as:

$$\frac{z_V}{R} = -\lambda_i \phi - EG \tag{3}$$

where E is an envelop function given by:

$$E = A_0 \phi \exp(A_1 \phi) \quad \text{if } \phi \le 4\pi$$
$$\mathrm{E} = \mathrm{M}\phi + \mathrm{B}\,\text{if } \phi \ge 4\pi \tag{4}$$

and

$$G = \sum_{n=0}^{N} C_n \cos n\phi + D_n \sin n\phi \tag{5}$$

Here, A_0, A_1, M, B, C_n and D_n are all empirical constants listed in "Egolf, A. and Landgrebe, A. J., "Helicopter Rotor Wake Geometry and Its Influence in Forward Flight, Vol. I, NASA CR-3726."

Beddoes Prescribed wake Model:

In this model, the x and y coordinates of the tip vortex are prescribed from a rigid wake model, as shown above. Te wake skew angle χ is next determined. Next, a new constant E = $\chi/2$ is computed. Beddoes assumes that the vertical velocity at which a vortex filament descends is given by

$$\lambda_i = \lambda_0 \left[1 + E\frac{x}{R} - E\left|\frac{y}{R}\right|^3 \right] \tag{6}$$

if the filament is underneath the rotor disk. If the filament is downstream of the rotor disk, then it is assumed to move downwards at a velocity given by

$$\lambda_i = 2\lambda_0 \left[1 - E\left|\frac{y}{R}\right|^3 \right] \tag{7}$$

The vertical position of the vortex filament is thus given by:

$$\frac{z_V}{R} = -\mu_z \phi + \int_0^{\phi} \lambda d\phi \tag{9}$$

The above integral may be numerically evaluated. The integrand will depend on whether the filament is underneath the rotor disk, or downstream of the rotor disk. Ref: Beddoes, T. S., “A Wake Model for High Resolution Airloads,” Second International Conference on Basic Rotorcraft Research, 1985, Research triangle Park, North Carolina.