Power Consumption in Forward Flight

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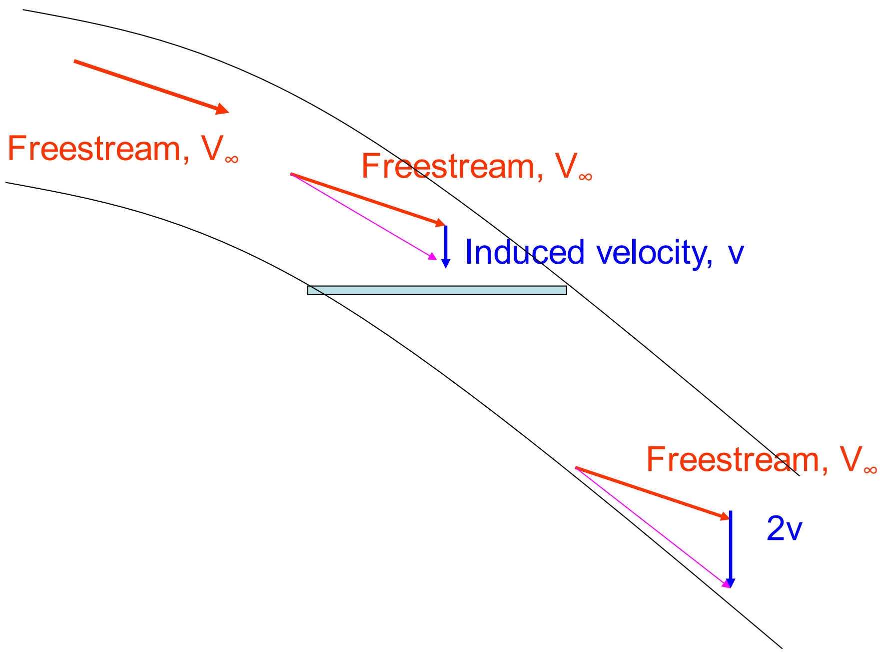
Induced Power

- Equals mass flow rate through the actuator disk times kinetic energy change of molecules per unit mass.
- Mass flow rate is density times disk area times velocity normal to disk
- Molecules entering the rotor disk have a kinetic energy per unit mass of ½ V_∞ squared



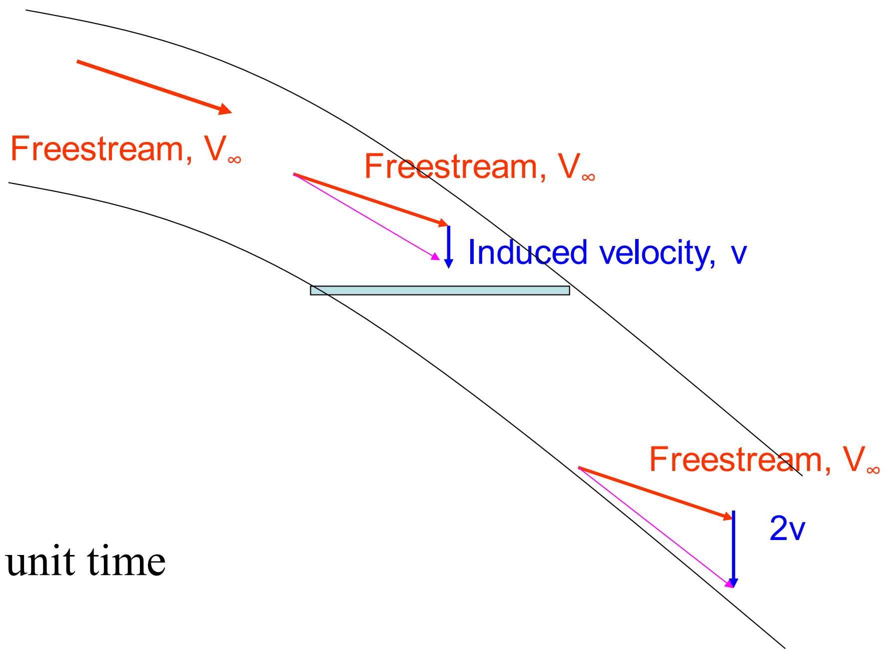
Kinetic energy flowing out per unit Mass = (Horizontal component)^2 + Vertical component^2 in the far wake.

Induced Power Consumed..

$$\text{Energy flowing in per unit time} = \frac{1}{2}\dot{m}\left[V_\infty^2\right]$$

$$\text{Energy Flowing Out per unit time} = \frac{1}{2}\dot{m}\left[\left(V_\infty \cos\alpha_{TPP}\right)^2 + \left(V_\infty \sin\alpha_{TPP} + 2\text{v}\right)^2\right]$$

$$= \frac{1}{2}\dot{m}\left[V_\infty^2 + 4\text{v}V_\infty \sin\alpha_{TPP} + 4\text{v}^2\right]$$



$$Power = (\text{Energy Flowing Out} - \text{Energy flowing in}) \text{ per unit time}$$

$$= 2\dot{m}\text{v}\left[V_\infty \sin\alpha_{TPP} + \text{v}\right]$$

$$= T\left[V_\infty \sin\alpha_{TPP} + \text{v}\right]$$

Power Consumption in Forward Flight

We can compute "Ideal Power" from Glauert's theory as Thrust times normal velocity at the rotor disk.

The actual power will include blade profile power, due to viscous drag on the blade. We will compute it later.

$$P = T\left(V_\infty \sin\alpha_{TPP} + \mathrm{v}\right) + \text{Profile Power}$$

$$\text{Recall } \mathrm{T}\sin\alpha_{\mathrm{TPP}} = D$$

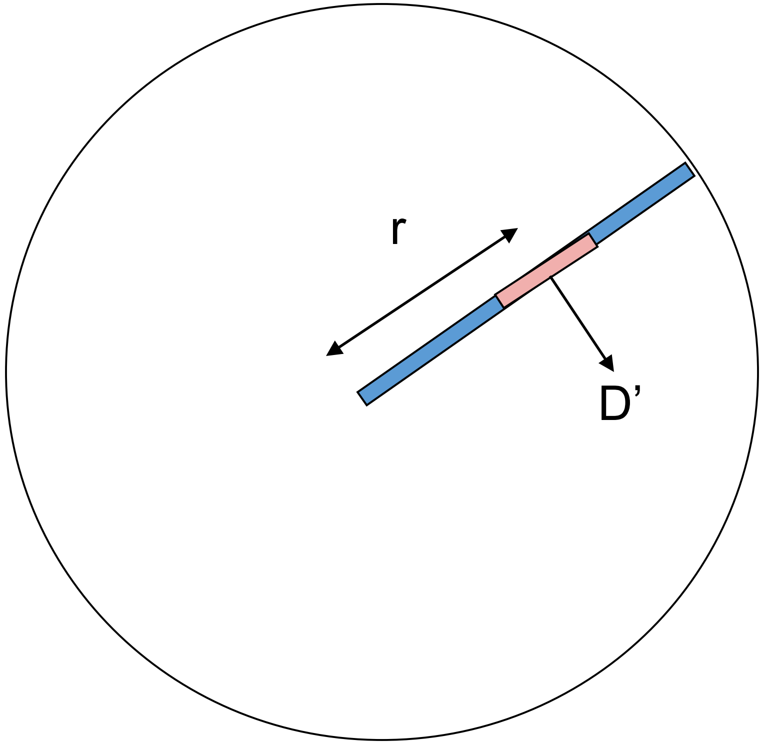
$$Thus,$$

$$P = T\mathrm{v} + \mathrm{D}V_\infty + \text{Profile Power}$$

Induced Power ($T\mathrm{v}$)

Parasite Power ($\mathrm{D}V_\infty$)

Profile Drag



$$D' = \frac{1}{2}\rho U_T^2 c C_{d,0}$$

$$where$$

$$U_T = \Omega r + V_\infty \sin\psi$$

We will assume chord c and drag coefficient C_{d0} are constant.

Integration of Profile Torque

$$Q_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R rD'dr\right\}\right]$$

Non-dimensionalize:

$$C_{Q,0} = \frac{Q}{\rho(\Omega R)^2 AR}$$

Final result:

$$C_{Q,0} = \frac{\sigma C_{d,0}}{8}\left[1+\mu^2\right]$$

Profile Power

$$Q_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R rD'dr\right\}\right]$$

$$P_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R U_T D'dr\right\}\right]$$

Non-dimensionalize:

$$C_{P,0} = \frac{P_0}{\rho(\Omega R)^3 A}$$

Final result:

$$C_{P,0} = \frac{\sigma C_{d,0}}{8}\left[1+3\mu^2\right]$$

Power Consumption in Level Flight

$P = T\text{v} + \text{DV}_\infty + \text{Blade Profile Power}$

The induced power Tv decreases with advance ratio μ, as discussed earlier.

$$\text{Parasite Power} = \text{DV}_\infty = \left[\frac{1}{2}\rho V_\infty^2 C_D S\right]\text{V}_\infty = \frac{1}{2}\rho V_\infty^3 C_D S$$

The parasite power increases as the cube of the velocity (or advance ratio μ) and dominates power consumption in high speed forward flight.

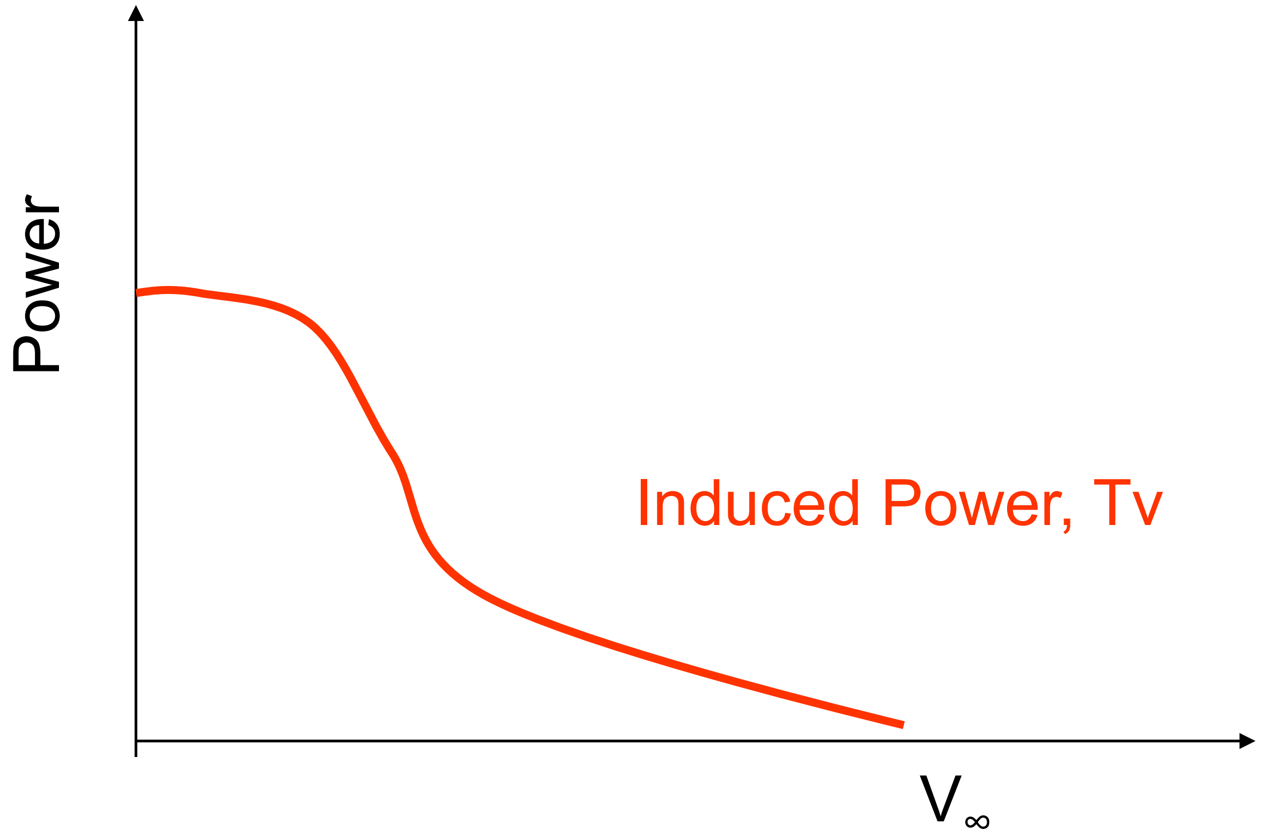
Here C_D is vehicle parasite drag coefficient, and S is the reference area C_D is based on.

Because there is no agreement on a common reference area, it is customary to supply the product $C_D S$.

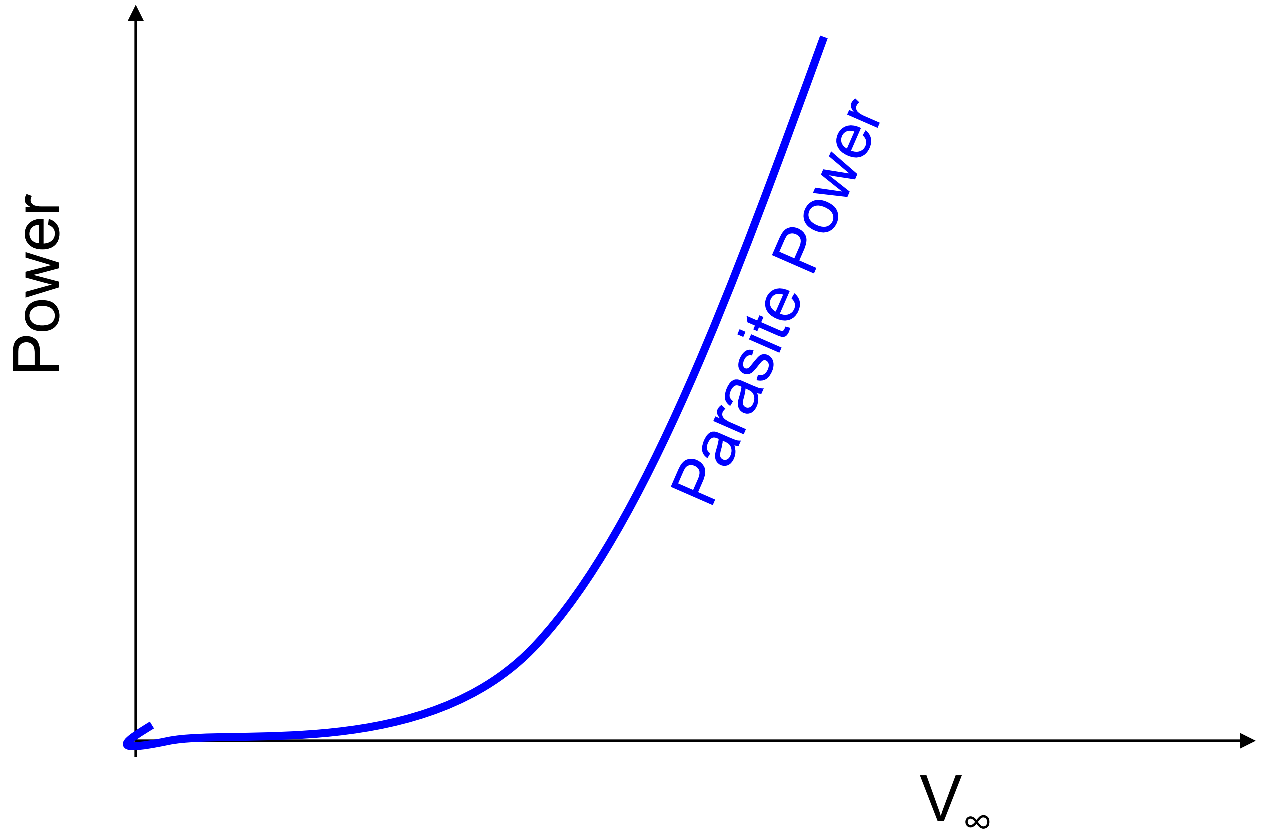
This product is called f, the equivalent flat plate area.

$$\text{Parasite Power} = \frac{1}{2}\rho V_\infty^3 f$$

Induced Power Consumption in Forward Flight



Parasite Power Consumption in Forward Flight

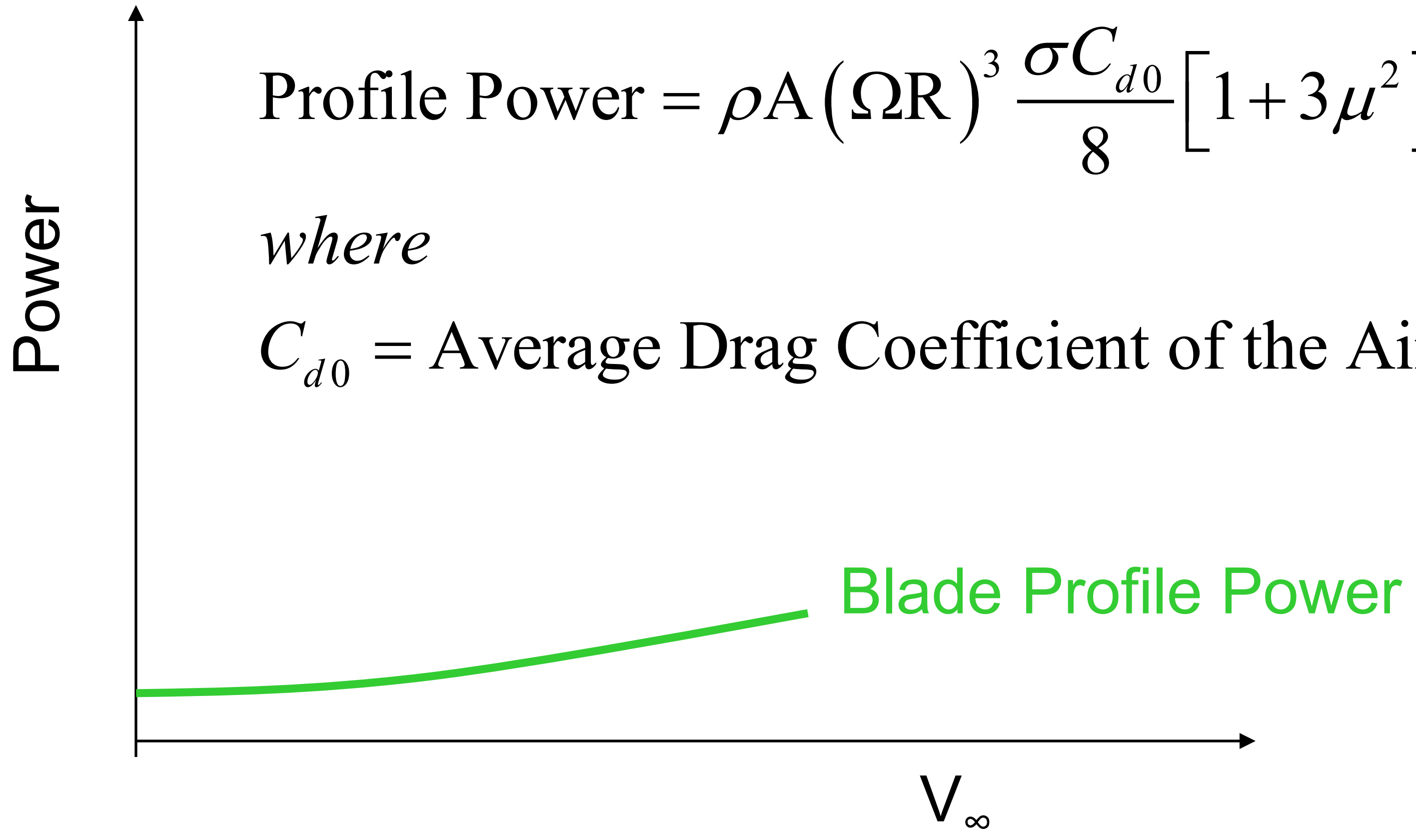


Profile Power Consumption in Forward Flight

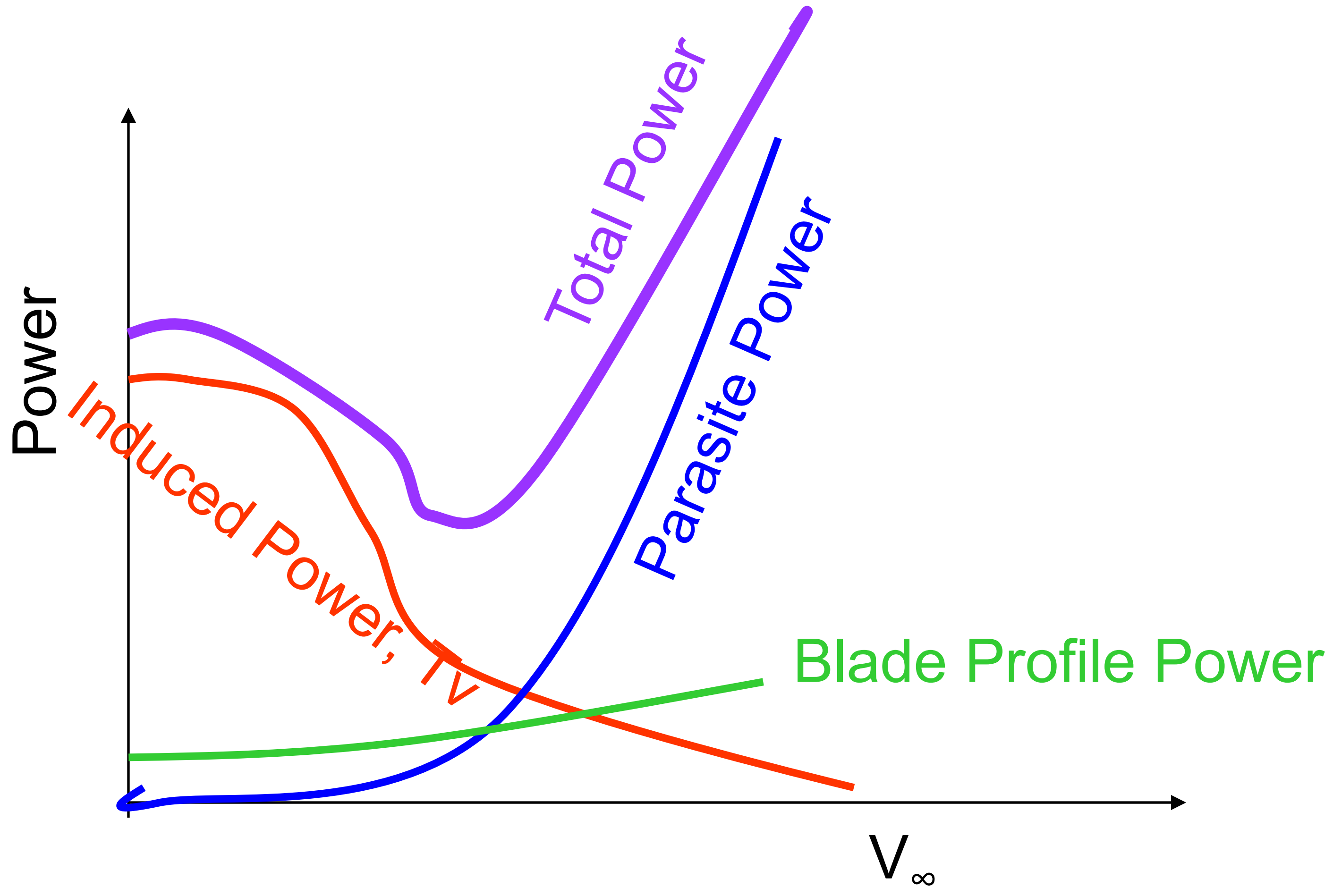
$$\text{Profile Power} = \rho A (\Omega R)^3 \frac{\sigma C_{d0}}{8}\left[1+3\mu^2\right]$$

where

C_{d0} = Average Drag Coefficient of the Airfoil



Power Consumption in Forward Flight



Non-Dimensional Expressions for Contributions to Power

$$\text{Recall}: \mathrm{C_P} = \frac{P}{\rho A(\Omega R)^3}$$

$$C_P = C_{P,i} + C_{P,parasite} + C_{P,0}$$

$$P_i = T\mathrm{v}$$

$$\mathrm{C_{P,i}} = \frac{T\mathrm{v}}{\rho A(\Omega R)^3} = \frac{T}{\rho A(\Omega R)^2} \times \frac{\mathrm{v}}{\Omega \mathrm{R}} = C_T \lambda_i$$

$$P_{parasite} = \frac{1}{2}\rho V_\infty^3 f \qquad C_{P,parasite} = \frac{\frac{1}{2}\rho V_\infty^3 f}{\rho A(\Omega R)^3} = \frac{1}{2}\frac{f}{A}\mu^3$$

$$\text{We will later show from blade element theory that } \mathrm{C_{P,0}} = \frac{\sigma C_{d,0}}{8}\left(1+3\mu^2\right)$$

$$\textit{Thus}, \quad C_P = C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d,0}}{8}\left(1+3\mu^2\right)$$

Empirical Corrections

- The performance theory above does not account for
  - Non-uniform inflow effects
  - Swirl losses
  - Tip Losses
- It also uses an average drag coefficient.
- To account for these, the power coefficient is empirically corrected.

Empirical Corrections

$$\text{Power Coefficent (Uncorrected)} = C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+3\mu^2\right]$$

Induced power — Parasite Power — Profile Power

$$\text{Power Coefficent (Corrected)} = \kappa C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+4.6\mu^2\right]$$

κ ↑ 1.15

Excess Power Determines Ability to Climb