Vortex Theory Based Inflow Models in Hover

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Hover Performance Prediction Methods

IV. Vortex Theory

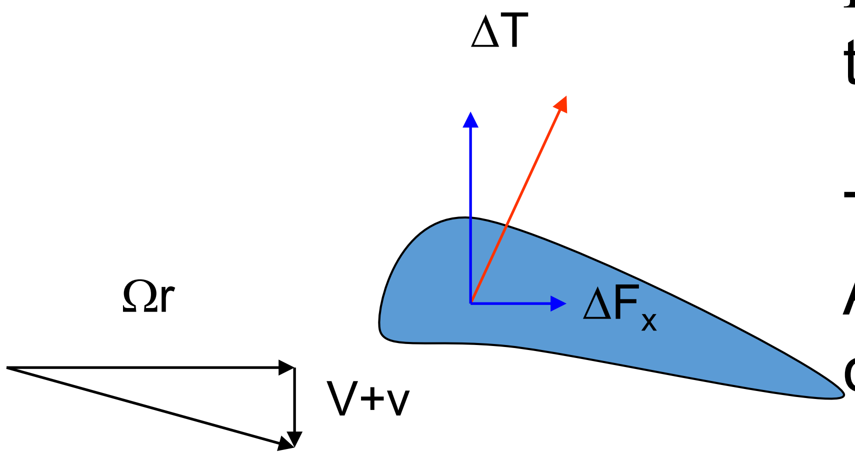
BACKGROUND

- Extension of Prandtl’s Lifting Line Theory
- Uses a combination of
  - Kutta-Joukowski Theorem
  - Biot-Savart Law
  - Empirical Prescribed Wake or Free Wake Representation of Tip Vortices and Inner Wake
- Robin Gray proposed the prescribed wake model in 1952.
- Landgrebe generalzied Gray’s model with extensive experimental data.
- Vortex theory was the extensively used in the 1970s and 1980s for rotor performance calculations, and is slowly giving way to CFD methods.

Background (Continued)

- Vortex theory addresses some of the drawbacks of combined blade element-momentum theory methods, at high thrust settings (high C_T/σ).
- At these settings, the inflow velocity is affected by the contraction of the wake.
- Near the tip, there can be an upward directed inflow (rather than downward directed) due to this contraction, which increases the tip loading, and alters the tip power consumption.

Kutta-Joukowsky Theorem



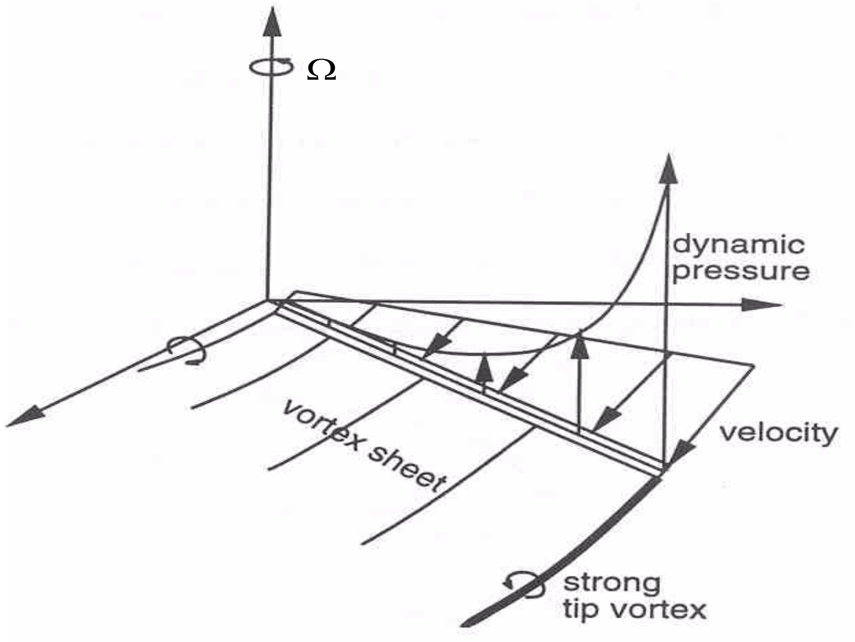
Γ : Bound Circulation surrounding the airfoil section.

This circulation is physically stored As vorticity in the boundary Layer over the airfoil

$$\Delta T = \rho\,(\Omega r)\,\Gamma$$

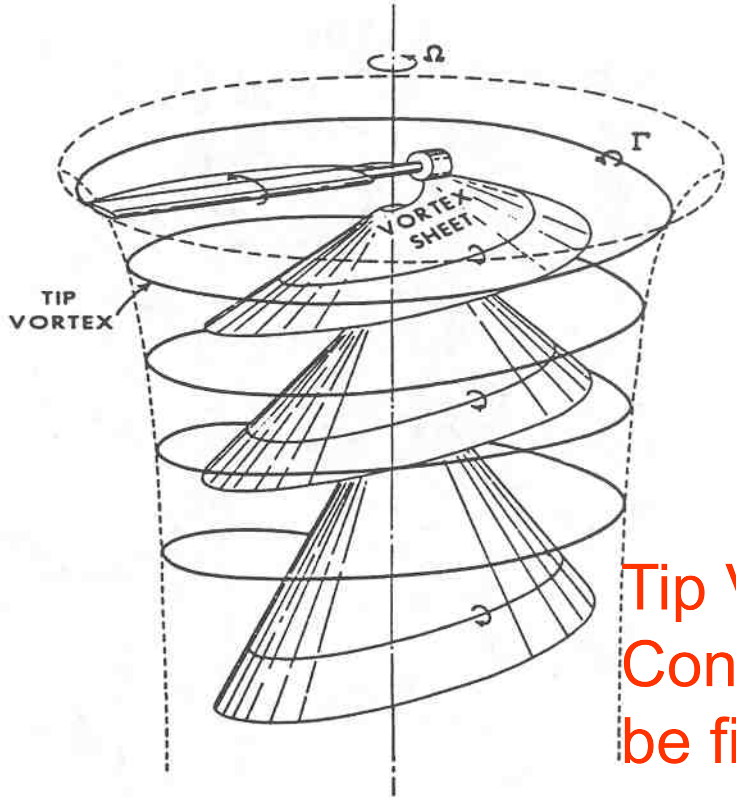
$$\Delta F_x = \rho\,(V+v)\,\Gamma$$

Representation of Bound and Trailing Vorticies



Since vorticity can not abruptly increase in space, trailing vortices develop. Some have clockwise rotation, others have counterclockwise rotation.

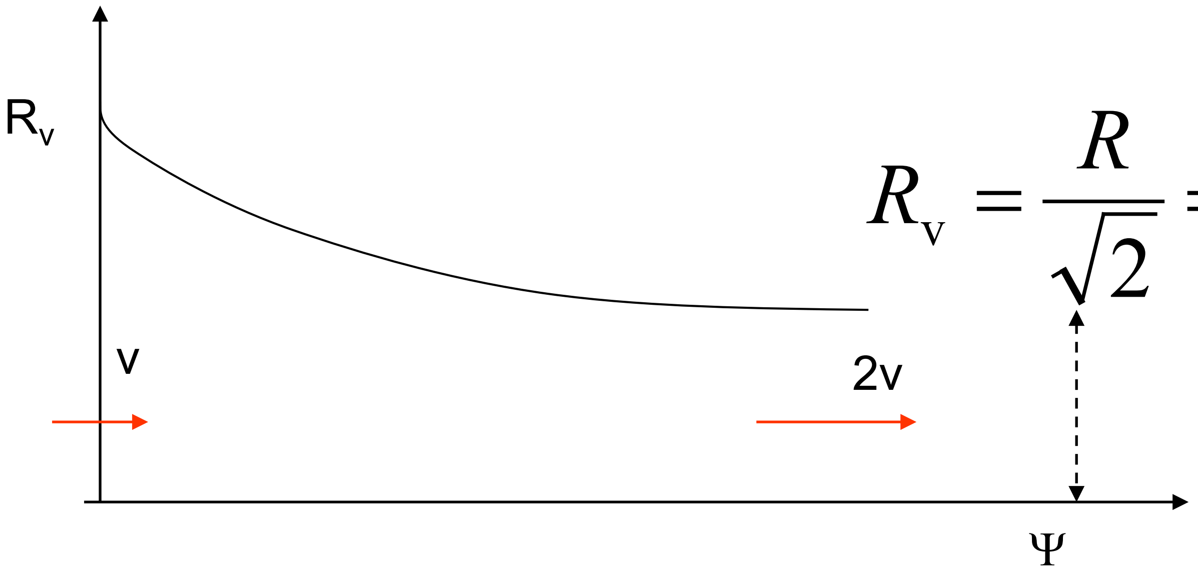
Robin Gray’s Conceptual Model



Tip Vortex has a Contraction that can be fitted with an exponential curve fit.

Inner wake descends at a near constant velocity. It descends faster near the tip than at the root.

Landgrebe's Curve Fit for the Tip Vortex Contraction



$$R_v = \frac{R}{\sqrt{2}} = 0.707R$$

Radial Contraction

Radial position of the tip vortex :

$$\frac{\mathrm{R}_{\mathrm{vortex}}}{\mathrm{R}} = A + (1 - A)e^{-\lambda\psi_{\mathrm{v}}}$$

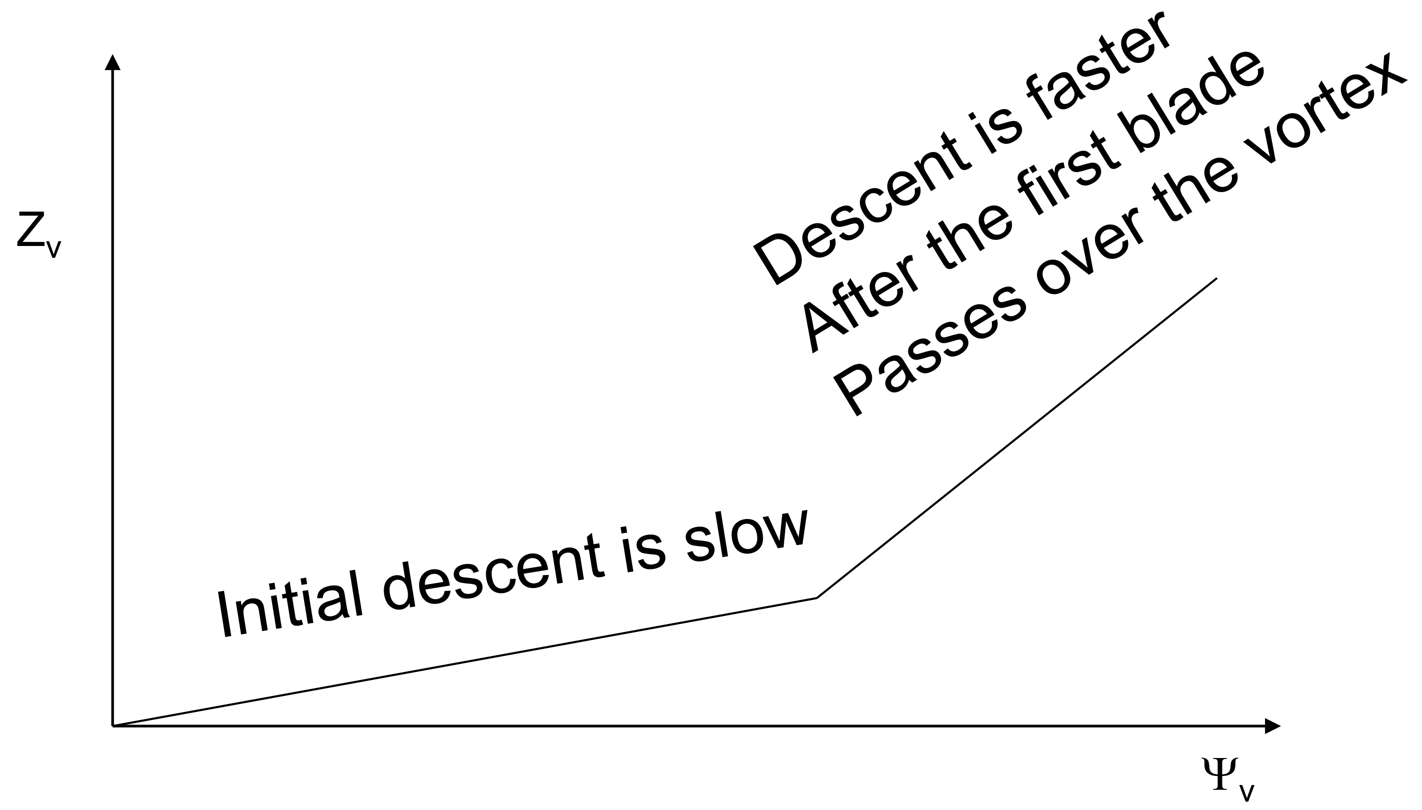
$$A = 0.78$$

$$\lambda = 0.145 + 27C_T$$

ψ_{v} = Vortex Age

= Azimuthal Position of the vortex Filament measured from the blade

Vertical Descent Rate



Landgrebe's Curve Fit for Tip Vortex Descent Rate

$$\frac{z_V}{R} = k_1 \psi_V \qquad 0 \le \psi_V \le \frac{2\pi}{b}$$

$$\frac{z_V}{R} = k_1 \frac{2\pi}{b} + \mathrm{k}_2\left(\psi_V - \frac{2\pi}{b}\right) \qquad \psi_V \ge \frac{2\pi}{b}$$

$$k_1 = -0.25\left[\frac{C_T}{\sigma} + 0.001\theta_{\text{twist, degrees}}\right]$$

$$k_2 = -\sqrt{C_T} - 0.01\sqrt{C_T}\,\theta_{\text{twist, degrees}}$$

$\theta_{\text{twist,degrees}}$: Blade twist=Tip Pitch angle – Root Pitch Angle
This quantity is usually negative.

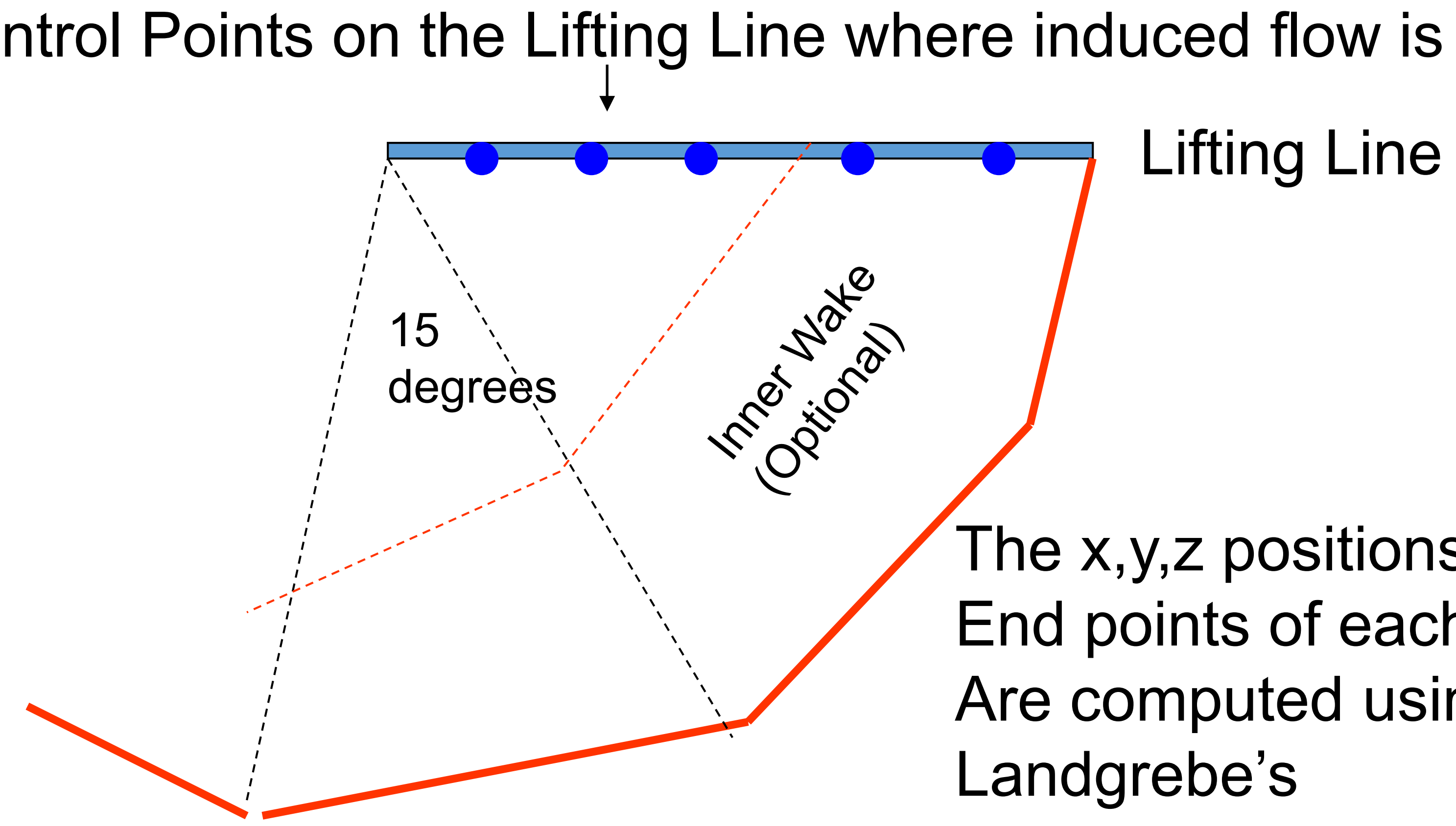
Circulation Coupled Wake Model

- Landgrebe's earlier curve fits (1972) were based on the thrust coefficient, blade twist (change in the pitch angle between tip and root, usually negative).
- He subsequently found (1977) that better curve fits are obtained if the tip vortex trajectory is fitted on the basis of peak bound circulation, rather than C_T/σ.

Tip Vortex Representation in Computational Analyses

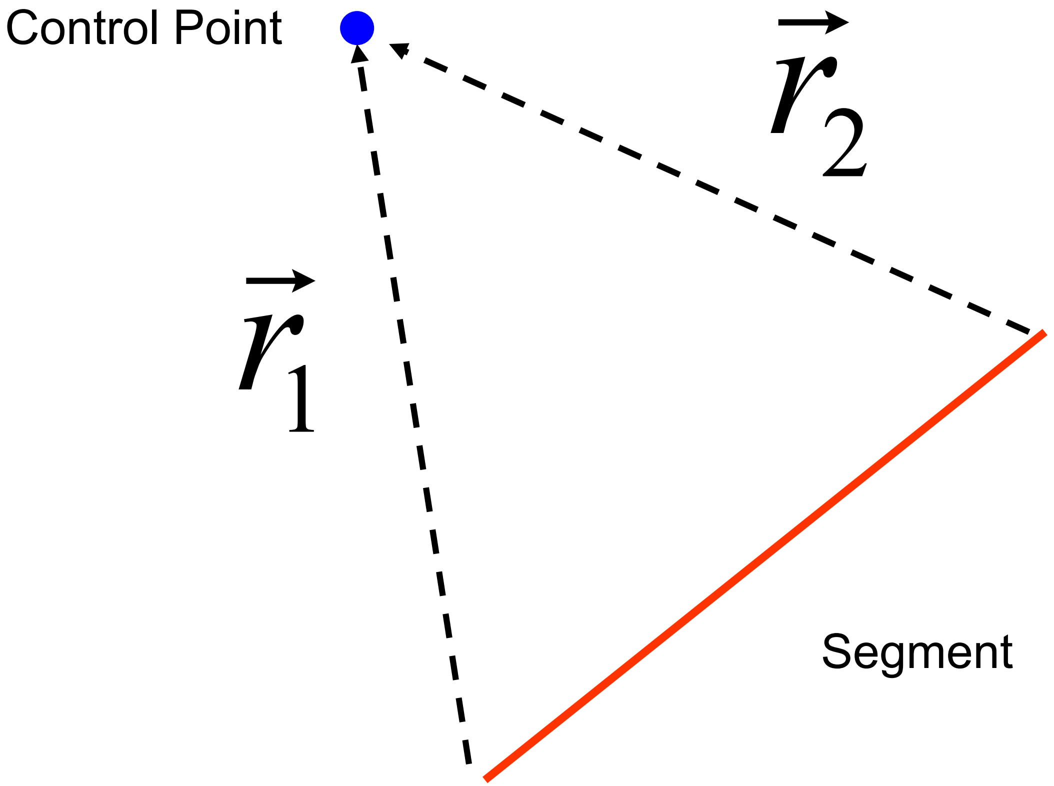
- The tip vortex is a continuous helical structure.
- This continuous structure is broken into piecewise straight line segments, each representing 15 degrees to 30 degrees of vortex age.
- The tip vortex strength is assumed to be the maximum bound circulation. Some calculations assume it to be 80% of the peak circulation.
- The vortex is assumed to have a small core of an empirically prescribed radius, to keep induced velocities finite.

Tip Vortex Representation



The x,y,z positions of the
End points of each segment
Are computed using
Landgrebe's
Prescribed Wake Model

Biot-Savart Law



Biot-Savart Law (Continued)

$$\vec{V}_{induced} = \frac{\Gamma}{4\pi} \vec{r}_1 \times \vec{r}_2 \frac{(r_1 + r_2)\left(1 - \vec{r}_1 \bullet \vec{r}_2 \big/ r_1 r_2\right)}{(r_1 r_2)^2 - (\vec{r}_1 \bullet \vec{r}_2)^2 + r_c^2\left(r_1^2 + r_2^2 - 2\vec{r}_1 \bullet \vec{r}_2\right)}$$

Core radius used to keep
Denominator from going to zero.

Overview of Vortex Theory Based Computations (Code supplied)

- Compute inflow using BEM first, using Biot-Savart law during subsequent iterations.
- Compute radial distribution of Loads.
- Convert these loads into circulation strengths. Compute the peak circulation strength. This is the strength of the tip vortex.
- Assume a prescribed vortex trajectory.
- Discard the induced velocities from BEM, use induced velocities from Biot-Savart law.
- Repeat until everything converges. During each iteration, adjust the blade pitch angle (trim it) if CT computed is too small or too large, compared to the supplied value.

Free Wake Models

- These models remove the need for empirical prescription of the tip vortex structure.
- We march in time, starting with an initial guess for the wake.
- The end points of the segments are allowed to freely move in space, convected the self-induced velocity at these end points.
- Their positions are updated at the end of each time step.

Free Wake Trajectories (Calculations by Leishman)