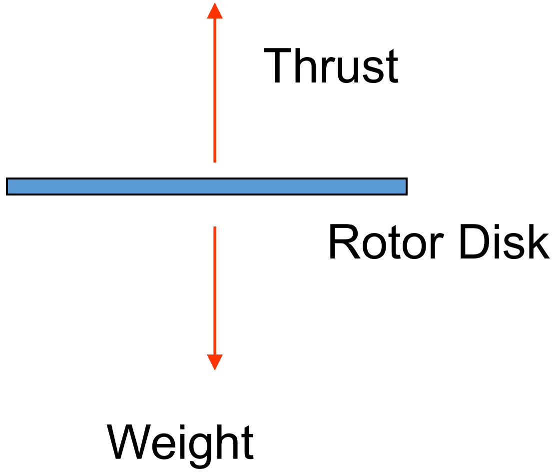
Steady, Level Forward Flight

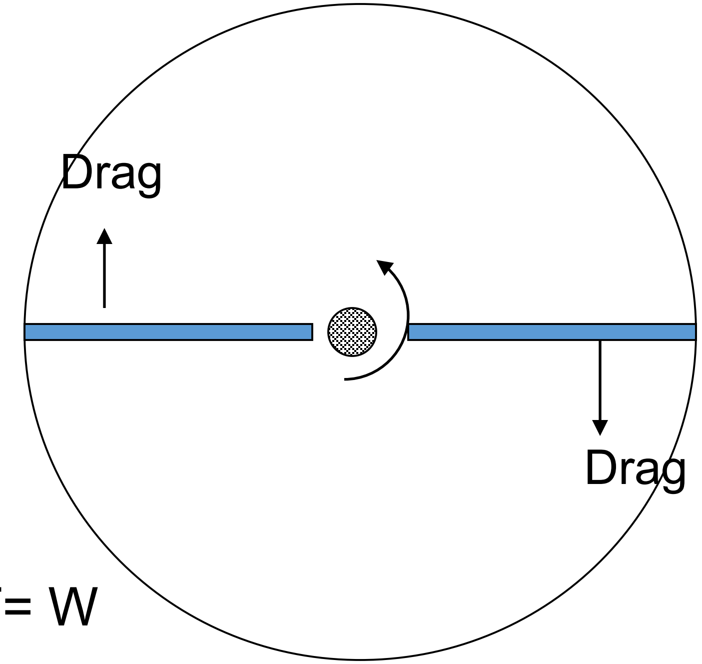
Inflow Model

Inflow Model

- To start this effort, we will need a very simple inflow model.
- A model proposed by Glauert is used.
- This model is phenomenological, not mathematically well founded.
- It gives reasonable estimates of inflow velocity at the rotor disk, and is a good starting point.
- It also gives the correct results for an elliptically loaded wing.

Force Balance in Hover



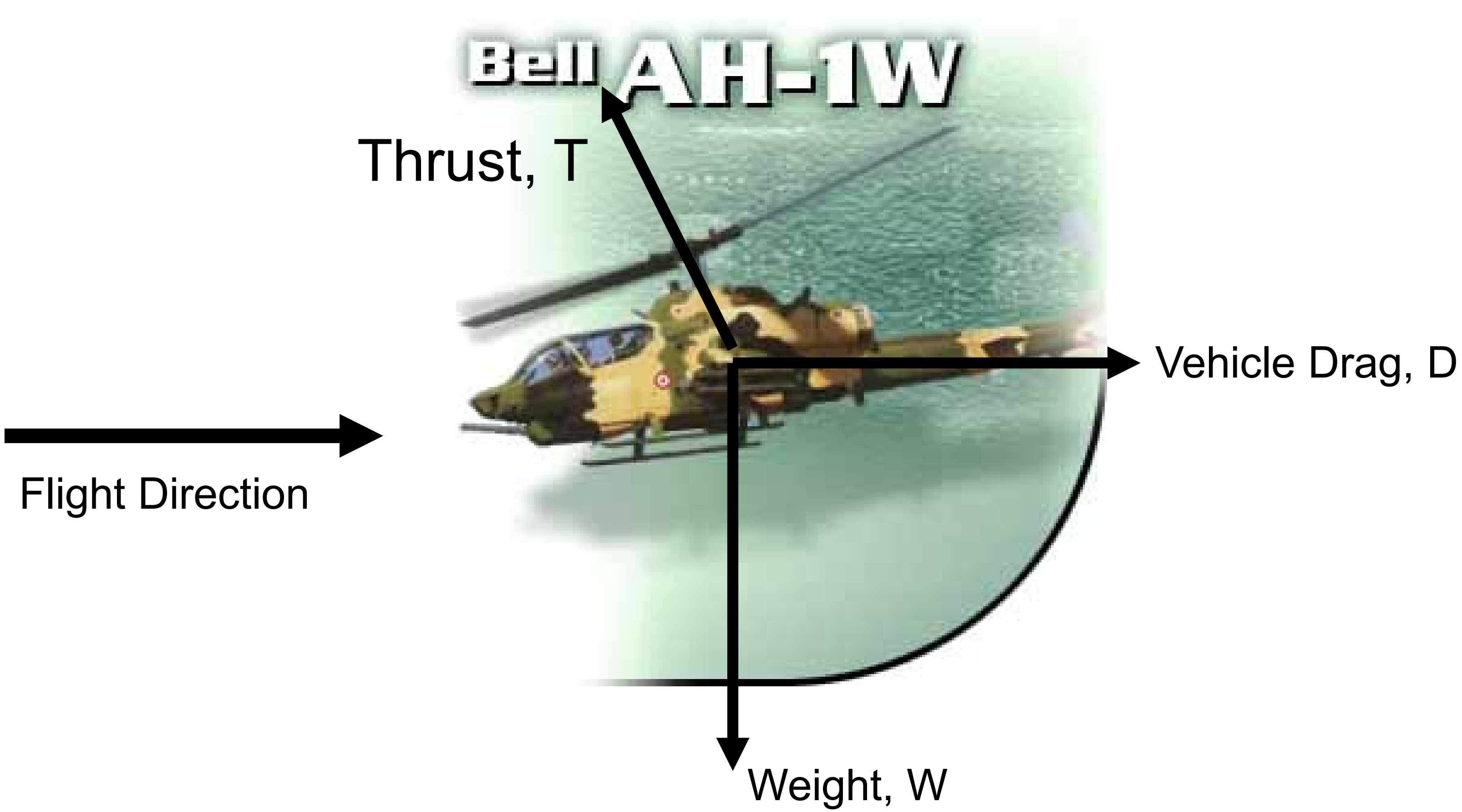


In hover, T= W

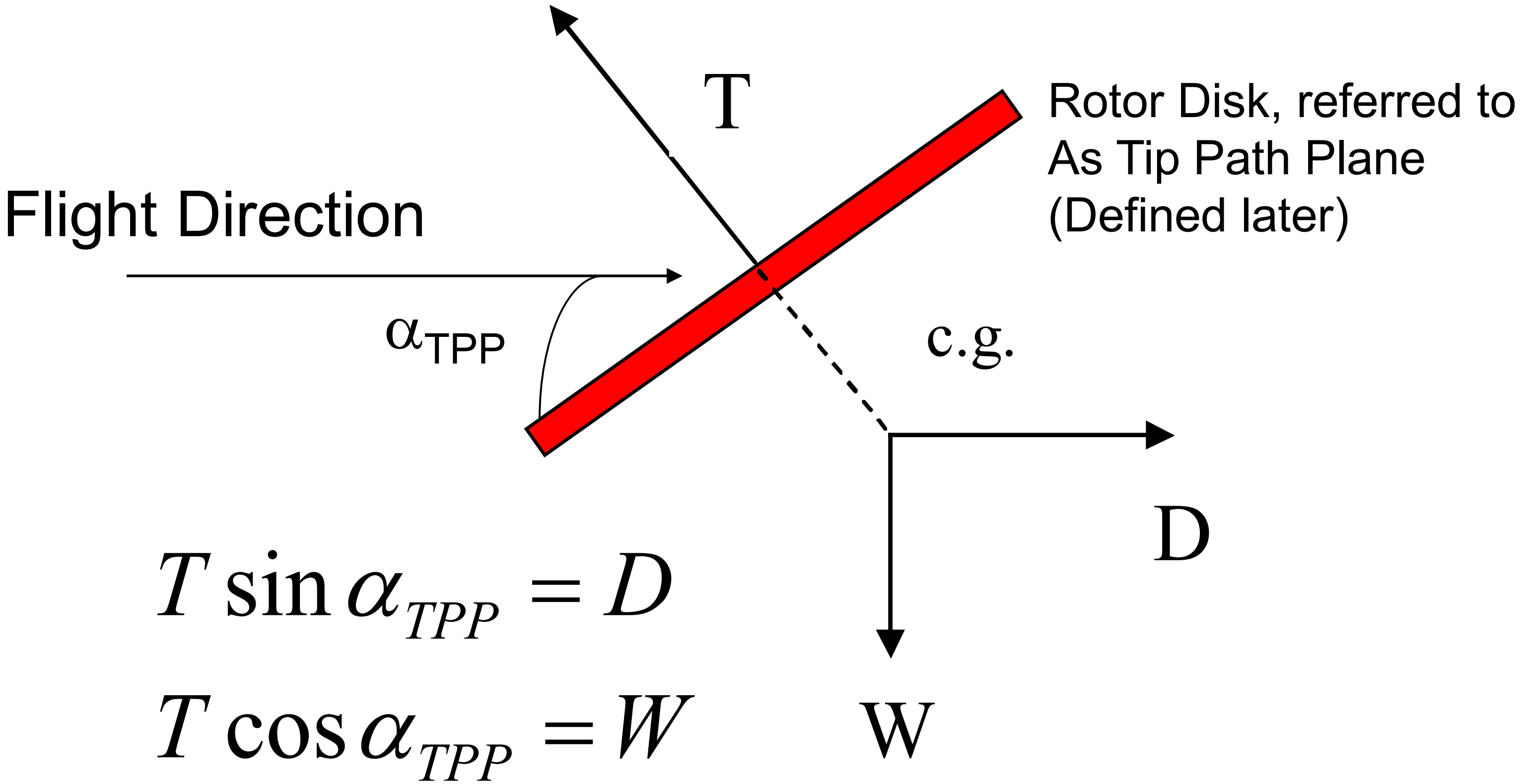
That is all!

No net drag, or side forces.
The drag forces on the individual blades
Cancel each other out, when summed up.

Force Balance in Forward Flight



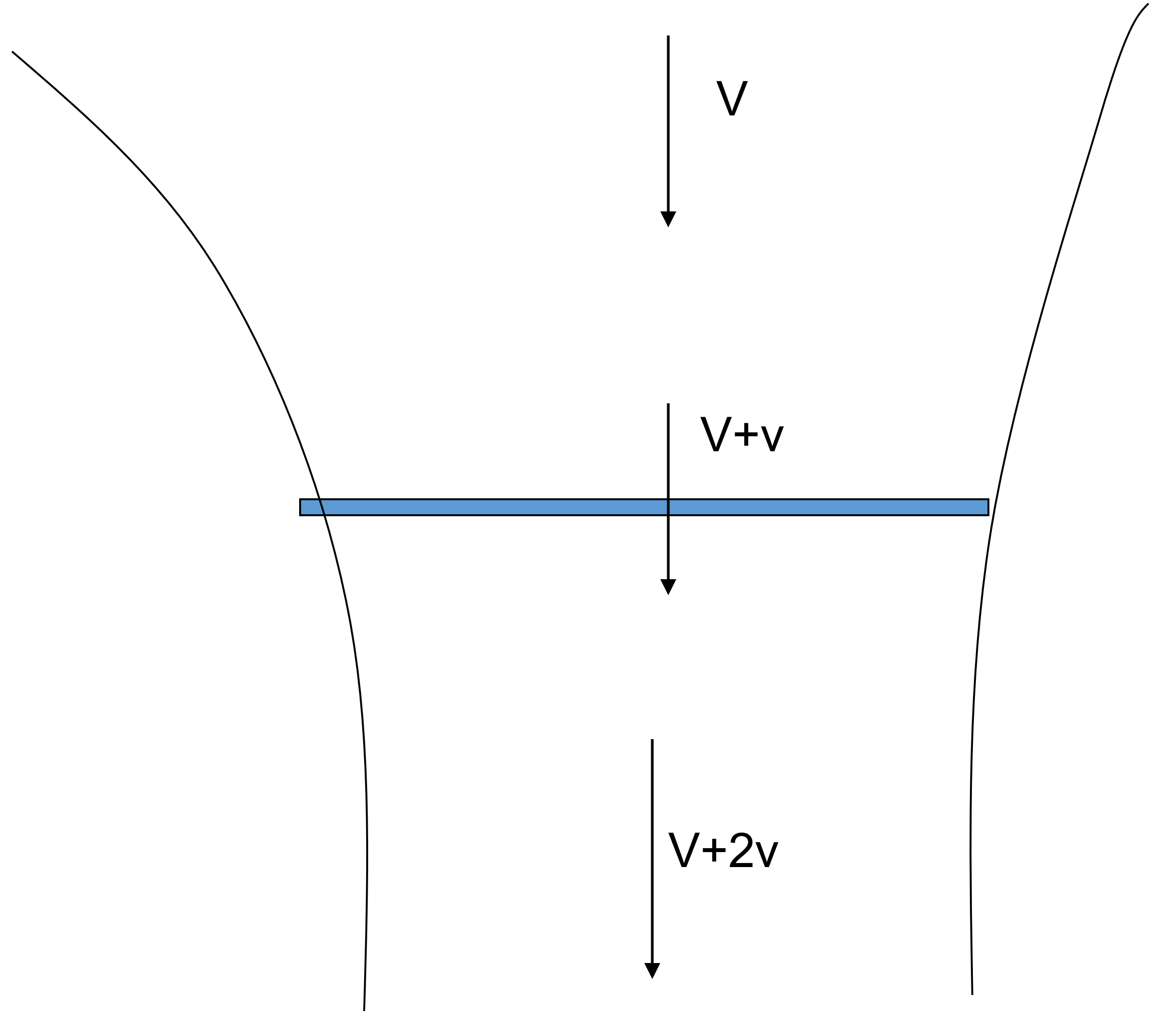
Simplified Picture of Force Balance



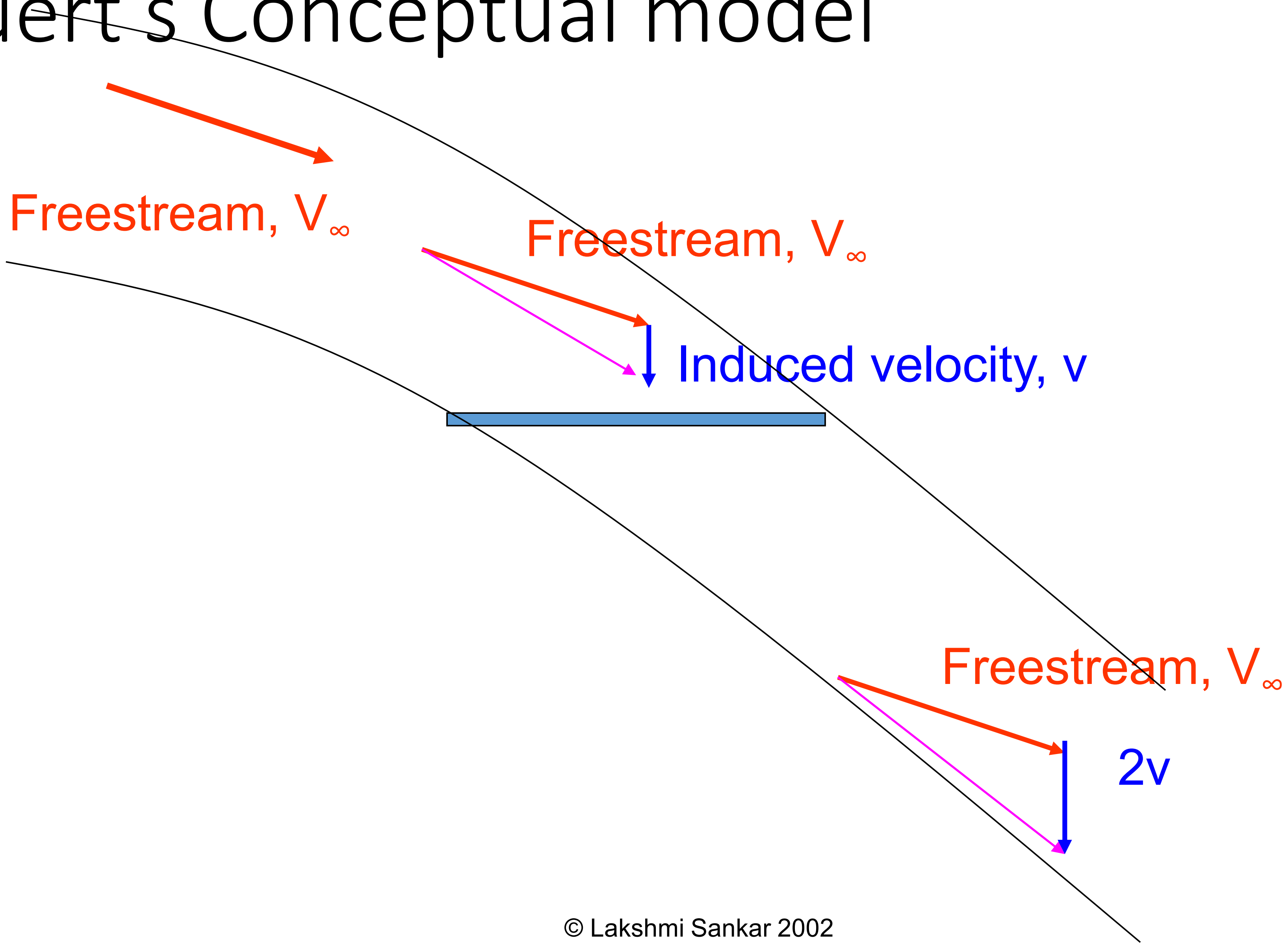
$$T \sin \alpha_{TPP} = D$$

$$T \cos \alpha_{TPP} = W$$

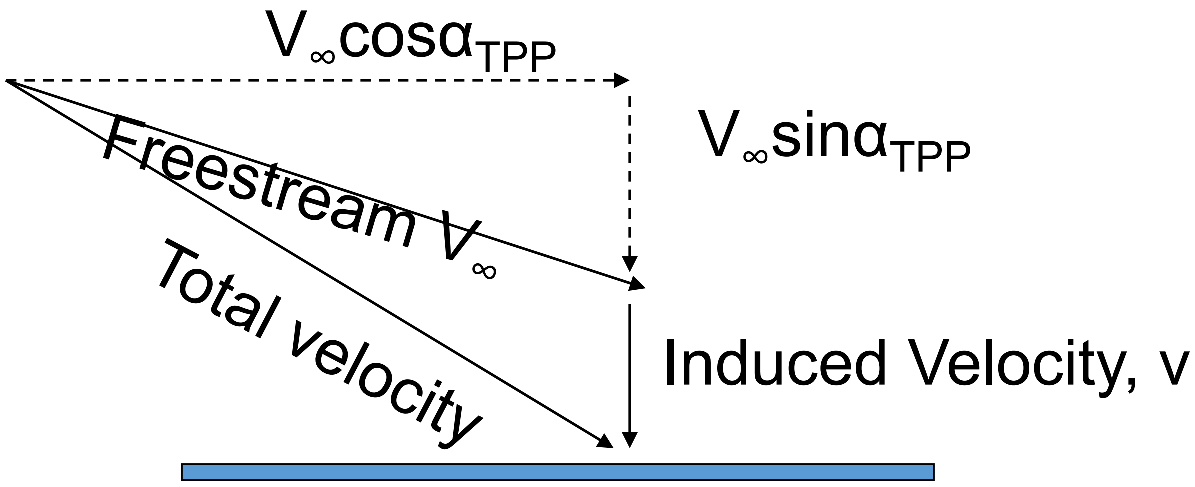
Recall the Momentum Model



Glauert's Conceptual model



Total Velocity at the Rotor Disk



$$\text{Total Velocity} = \sqrt{\left(V_\infty \cos\alpha_{TPP}\right)^2 + \left(V_\infty \sin\alpha_{TPP} + v\right)^2}$$

Relationship between Thrust and Velocities

In the case of hover and climb, recall

Thrust = (mass flow rate) *change in induced velocity

$$T = \rho A (V+v) (2v)$$

Mass flow rate — $(V+v)$

Change in Induced Velocity — $(2v)$

Glauert used the same analogy in forward flight.

In forward flight..

$$T = (2\mathrm{v})\, \rho A \sqrt{\left(V_\infty \cos \alpha_{TPP}\right)^2 + \left(V_\infty \sin \alpha_{TPP} + \mathrm{v}\right)^2}$$

This is a non-linear equation for induced velocity v, which must be iteratively solved for a given T, A, and tip path plane angle α_{TPP}

It is convenient to non-dimensionalize all quantities.

Non-Dimensional Forms

$$C_T = \frac{T}{\rho A(\Omega R)^2}$$

$$\frac{\text{Edgewise Freestream Component}}{\text{Tip Speed}} = \frac{V_\infty \cos\alpha_{TPP}}{\Omega R} \approx \frac{V_\infty}{\Omega R}$$

is called advance ratio, μ

Non-dimensinal inflow ratio, $\lambda_i = \dfrac{v}{\Omega R}$

Glauert equation in non-dimensional form becomes

$$C_T = 2\lambda_i \sqrt{\mu^2 + (\mu \tan\alpha_{TPP} + \lambda_i)^2}$$

Approximate Form at High Speed Forward Flight

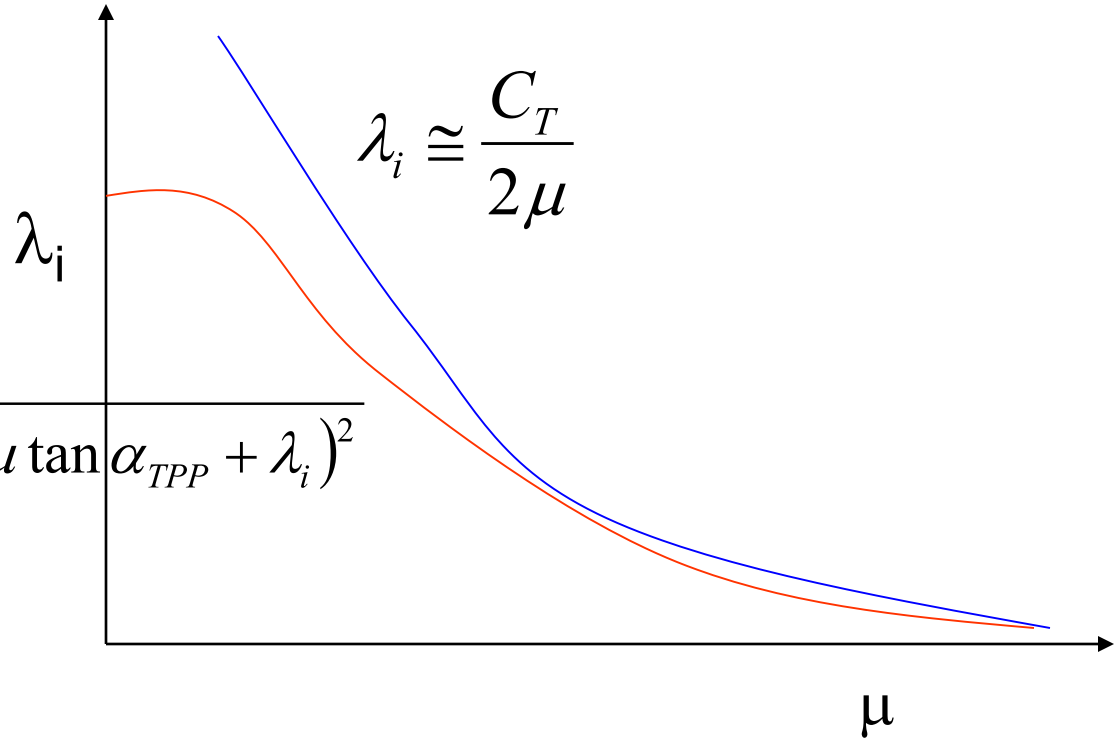
If advance ratio μ is higher than 0.2,
and if tip path plane angle is small,
μ far exceeds inflow ratio λ_i so that

$$C_T = 2\lambda_i \sqrt{\mu^2 + \left(\mu \tan \alpha_{TPP} + \lambda_i\right)^2}$$
$$\cong 2\mu\lambda_i$$

$$\lambda_i = \frac{C_T}{2\mu}$$

In practice, advance ratio μ seldom exceeds 0.4, because of limitations associated with forward speed.

Variation of Non-Dimensional Inflow with Advance Ratio



$$\lambda_i \cong \frac{C_T}{2\mu}$$

$$C_T = 2\lambda_i\sqrt{\mu^2 + \left(\mu \tan \alpha_{TPP} + \lambda_i\right)^2}$$

Notice that inflow velocity rapidly decreases with advance ratio.