Forward Flight

Angle of Attack of the Airfoil

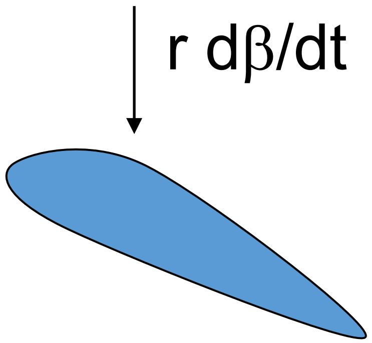
And

Sectional Load Calculations

The angle of attack of an airfoil depends on

- Pilot input: collective θ_0 and cyclic pitch θ_{1c}, and θ_{1s}
- How the blade is twisted
  - $\theta = \theta_0 + \theta_{twist}\, r/R + \theta_{1c} \cos\psi + \theta_{1s} \sin\psi$
- Inflow due to freestream component, and induced inflow
- Velocity of the air normal to blade chord, caused by the blade flapping
- Anhedral and dihedral effects due to coning of the blades.
- The first two bullets are self-evident.
- Let us look at the other contributors to the angle of attack.

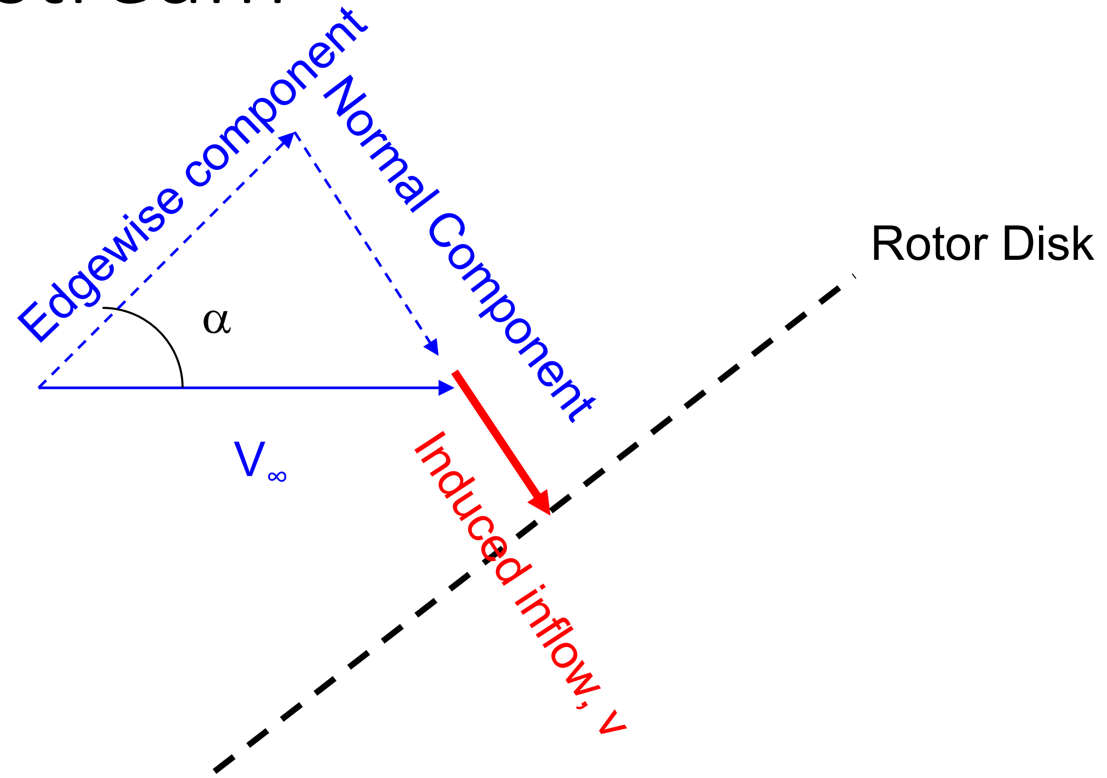
Blade Flapping Effect

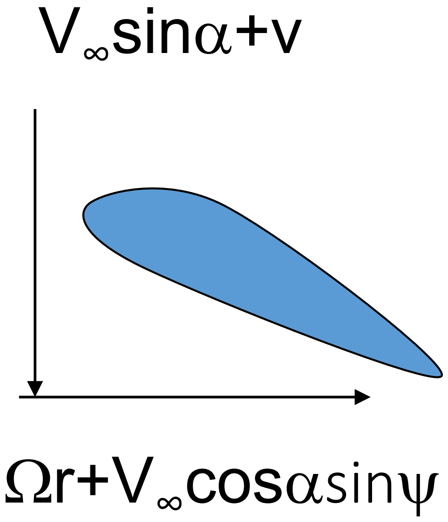


The blade section flaps up at a velocity r$\dot{\beta}$.

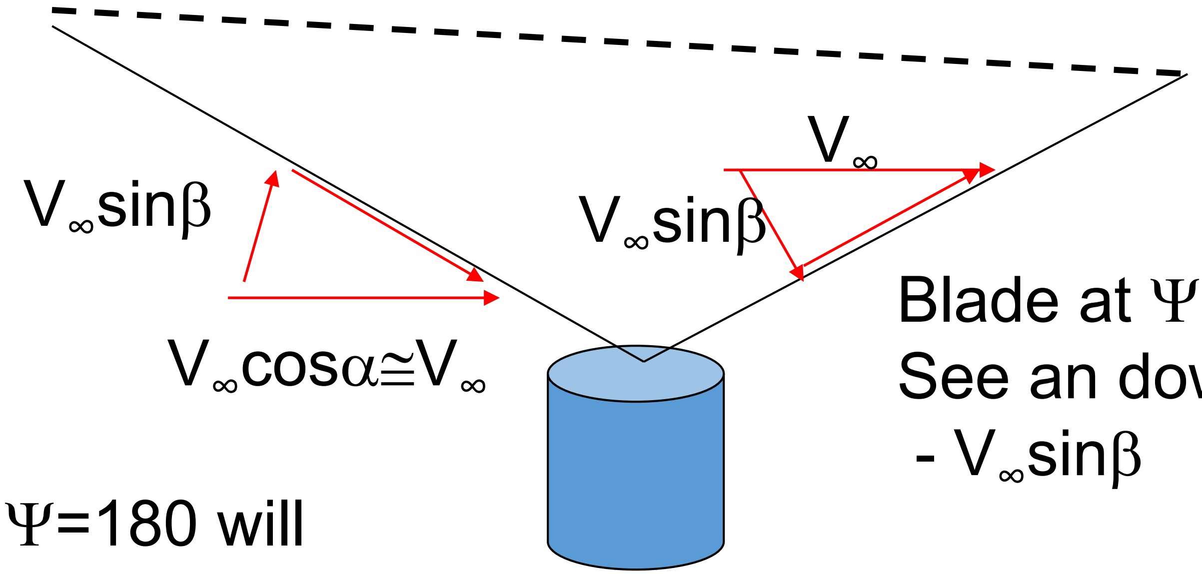
The airfoil thinks it is experiencing a downwash of equal magnitude.

Angle of Attack caused by the Inflow and Freestream





Anhedral/Dihedral Effect

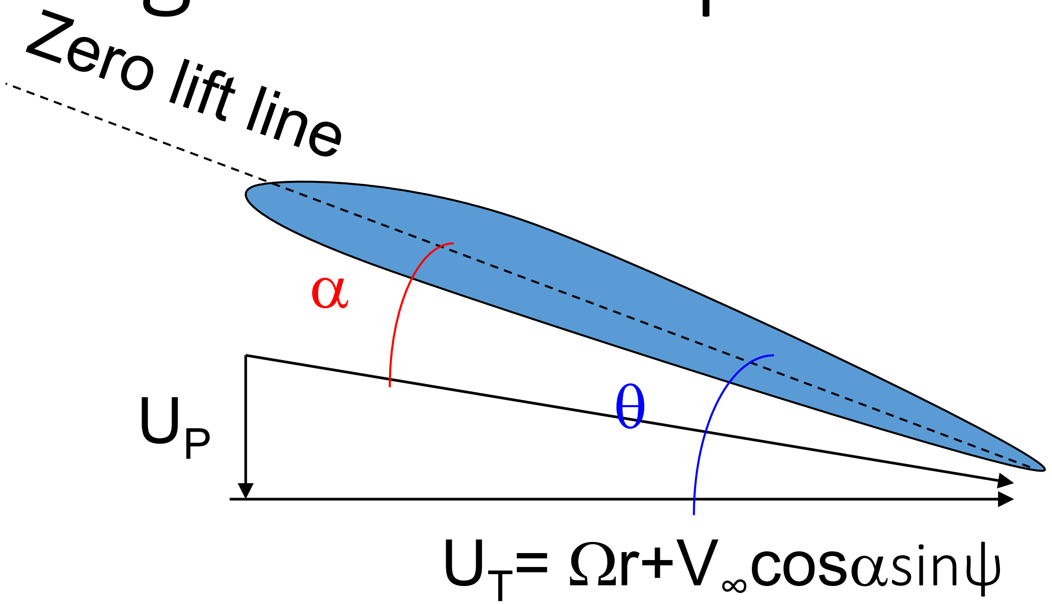


Blade at Ψ=0 will
See an downwash equal to
- $V_\infty \sin\beta$

Blade at Ψ=180 will
See an upwash equal to $V_\infty \sin\beta$

Blade at any Ψ will see an upwash equal to
- $V_\infty \sin\beta \cos \Psi$

Summing them all up..



$$\alpha = \theta - \tan^{-1}\left[\frac{U_P}{U_T}\right] \cong \theta - \frac{U_P}{U_T}$$

$$U_P = V_\infty \sin\alpha + \mathrm{v} + \mathrm{r}\dot{\beta} + \mathrm{V}_\infty \cos\psi \sin\beta$$

Small Angle of Attack Assumptions

- The angle of attack a (which is the angle between the freestream and the rotor disk) is small.
- The cyclic and collective pitch angles are all small.
- The coning and flapping angles are all small.
- $\cos(\alpha) = \cos(\beta) = \cos(\theta) \sim 1$
- $\sin(\alpha) \sim \alpha$, $\sin(\beta) \sim \beta$, $\sin(\theta) \sim \theta$

Angle of Attack

$$\alpha_{effecive} = \theta - \arctan\left(\frac{U_P}{U_T}\right) \cong \theta - \frac{U_P}{U_T} = \frac{U_T\theta - U_P}{U_T}$$

$$\theta = \theta_0 + \theta_{tw}\frac{r}{R} + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi$$

$$U_T = \Omega r + V_\infty \cos\alpha_s \sin\psi \cong \Omega r + V_\infty \sin\psi$$

$$U_P = \Omega R\lambda + r\dot{\beta} + V_\infty \cos\alpha_s \beta\cos\psi$$

$$\cong \Omega R\lambda_s + r\dot{\beta} + V_\infty \beta\cos\psi$$

Subscript s: All angles are in the shaft plane

Angle of attack (continued)

$$U_T\theta - U_P$$
$$= \left(\Omega r + V\sin\psi\right)\cdot\left(\theta_0 + \theta_{tw}\frac{r}{R} + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi\right)$$
$$-\left(\Omega R\lambda_s + r\dot{\beta} + V_\infty\beta\cos\psi\right)$$

$$\dot{\beta} = \frac{d\beta}{dt} = \left(\frac{d\beta}{d\psi}\right)\cdot\left(\frac{d\psi}{dt}\right) = \Omega\frac{d\beta}{d\psi}$$
$$= \Omega\left(\beta_{1s}\cos\psi - \beta_{1c}\sin\psi\right)$$

Angle of Attack (Continued)

After some minor algebra,

$$U_T\theta - U_P = \Omega r\left\{\theta_0 + \theta_{tw}\frac{r}{R} + \left(\theta_{1c} - \beta_{1s}\right)\cos\psi + \left(\theta_{1s} + \beta_{1c}\right)\sin\psi\right\}$$

$$V_\infty\left(\theta_0 + \theta_{tw}\frac{r}{R}\right)\sin\psi + V_\infty\left(\theta_{1c} - \beta_{1s}\right)\cos\psi\sin\psi +$$

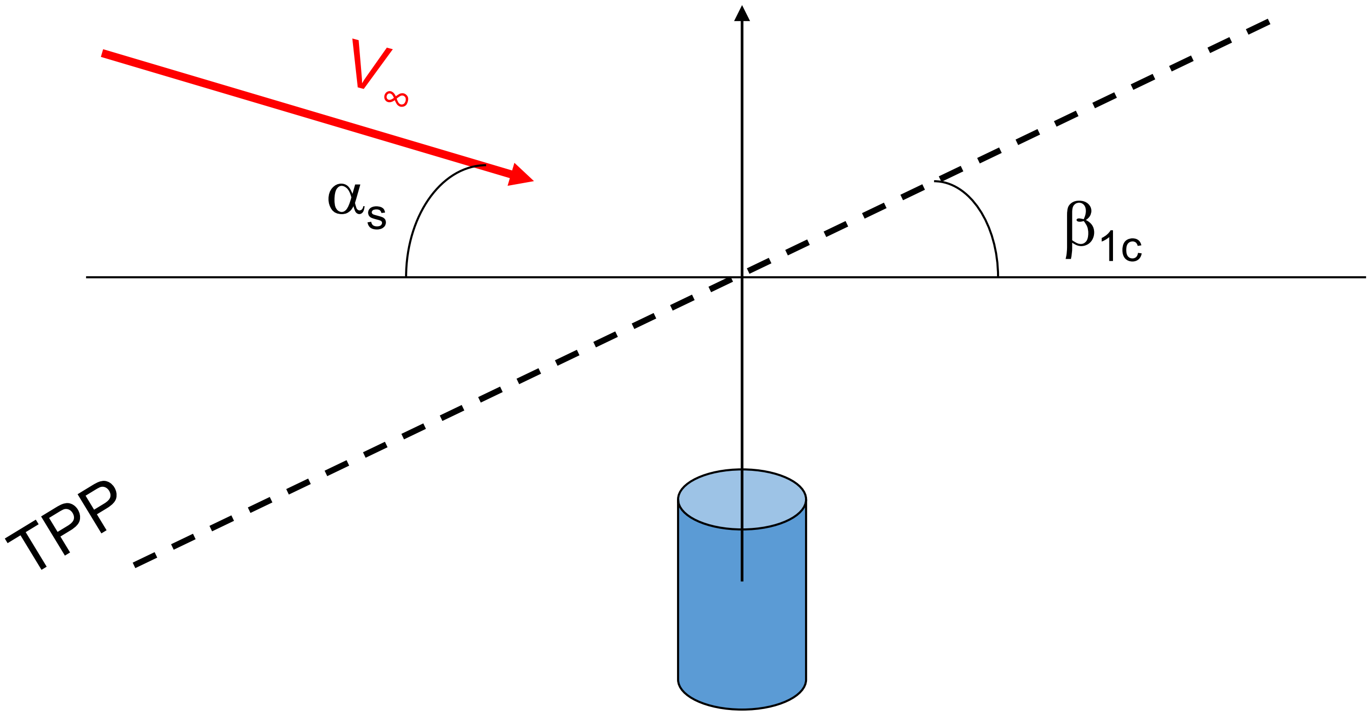
$$V_\infty\left(\theta_{1s} + \beta_{1c}\right)\sin^2\psi - V_\infty\beta_0\cos\psi - V_\infty\alpha_{TPP} - \mathrm{v}$$

Notice $\beta_{1c}+\theta_{1s}$ appears in pair, as pointed out earlier.

Also $\theta_{1c}-\beta_{1s}$ appears in pairs.

One degree of pitching is equivalent to one degree of Flapping.

Relationship between α_{TPP} and α_s



$$\alpha_{TPP}=\alpha_s+\beta_{1c}$$

Angle of Attack (Concluded)

$$\alpha_{effective} = \frac{U_T\theta - U_P}{U_T} = \frac{1}{U_T}\begin{bmatrix} \Omega r\left\{\theta_0 + \theta_{tw}\frac{r}{R} + \left(\theta_{1c} - \beta_{1s}\right)\cos\psi + \left(\theta_{1s} + \beta_{1c}\right)\sin\psi\right\} \\ V_\infty\left(\theta_0 + \theta_{tw}\frac{r}{R}\right)\sin\psi + V_\infty\left(\theta_{1c} - \beta_{1s}\right)\cos\psi\sin\psi \\ +V_\infty\left(\theta_{1s} + \beta_{1c}\right)\sin^2\psi - V_\infty\beta_0\cos\psi \\ -V_\infty\alpha_{TPP} - \mathrm{v} \end{bmatrix}$$

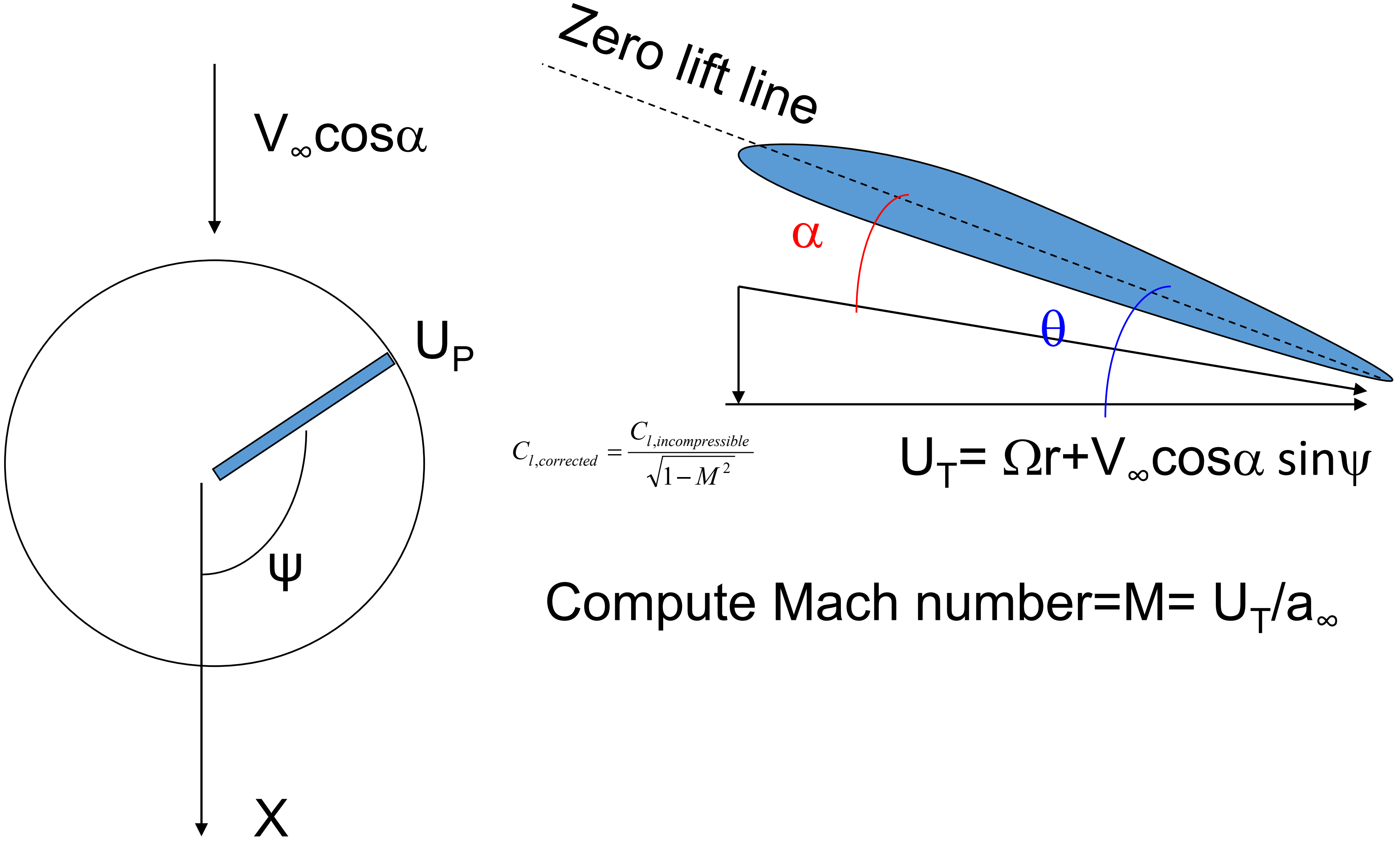
Calculation of Sectional Loads

- Once the angle of attack at a blade section is computed as shown in the previous slide, one can compute lift, drag, and pitching moment coefficients.
- This can be done in a number of ways. Modern rotorcraft performance codes (e.g. CAMRAD) give the user numerous choices on the way the force coefficients are computed.
  - Table Look-Up of C_ℓ and C_d vs. α as a function of local Mach number
    - This table is commonly called a C-81 table.
  - Unsteady aerodynamics corrections
  - Dynamic Stall corrections
  - Leading edge sweep corrections
- It is important to correct for dynamic stall effects in high speed forward flight to get the vibration levels of the vehicle correct.

Some choices for Computing Sectional Loads as a function of α

- In analytical work, it is customary to use $Cl=a\alpha$, $C_d=C_{d0}$ = constant, and $C_m= C_{mo}$, a constant. Here "a" is the lift curve slope, close to 2π.
- In simple computer based simulations using Excel or a program, these loads are corrected for compressibility using Prandtl-Glauert Rule.
- More sophisticated calculations will use C-81 tables, with corrections for the local sweep angle.

Prandtl-Glauert Rule



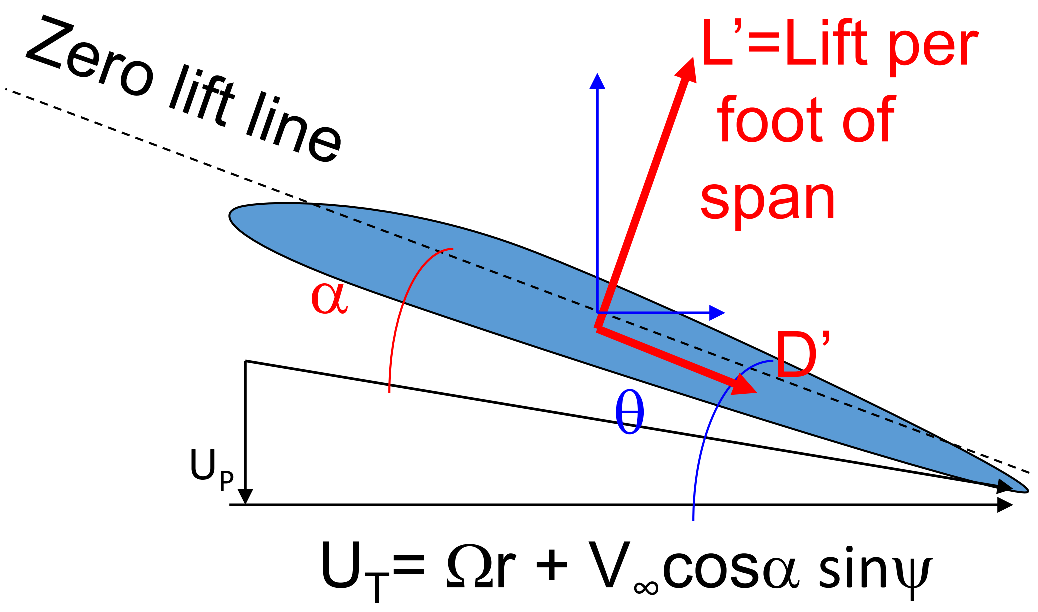
$$C_{l,corrected} = \frac{C_{l,incompressible}}{\sqrt{1-M^2}}$$

$U_T = \Omega r + V_\infty \cos\alpha \sin\psi$

Compute Mach number=M= U_T/a_∞

Calculation of Sectional Forces

- After C_l, C_d, C_m are found, one can find the lift, drag, and pitching moments per unit span.
- These loads are normal to, and along the total velocity, and must be rotated appropriately.



$$L' = \frac{1}{2}\rho U^2 c C_l$$

$$\cong \frac{1}{2}\rho c(U_T^2 + U_P^2) a \alpha_{eff}$$

$$\cong \frac{1}{2}\rho c a U_T \left(U_T \theta - U_P\right)$$

$$\alpha_{eff} = \theta - \frac{U_P}{U_T}$$