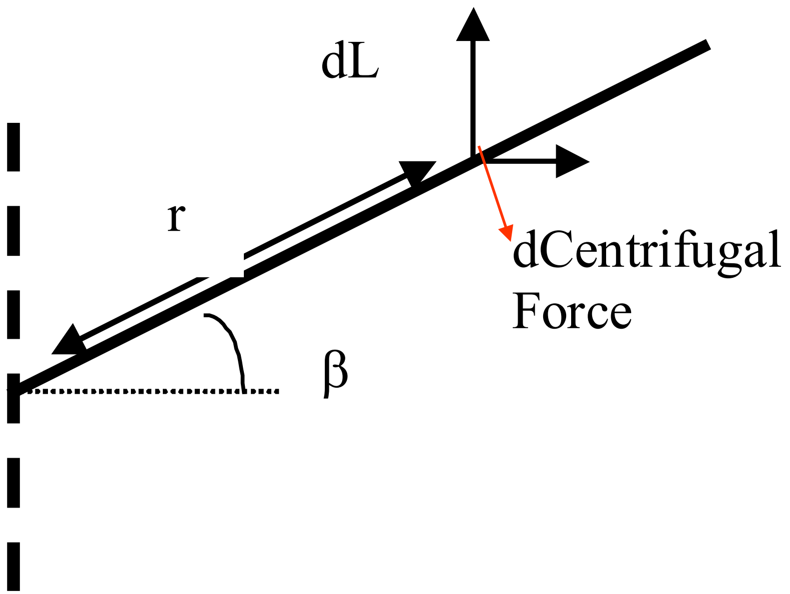
Forward Flight

Trimming the Rotor for Specified Hub Loads

Background

- In the previous sections, we developed expressions for sectional angle of attack, sectional loads, total thrust, torque, power, H-force and Y-force.
- These equations assumed that the pilot control settings and the blade flapping dynamics are known a priori.

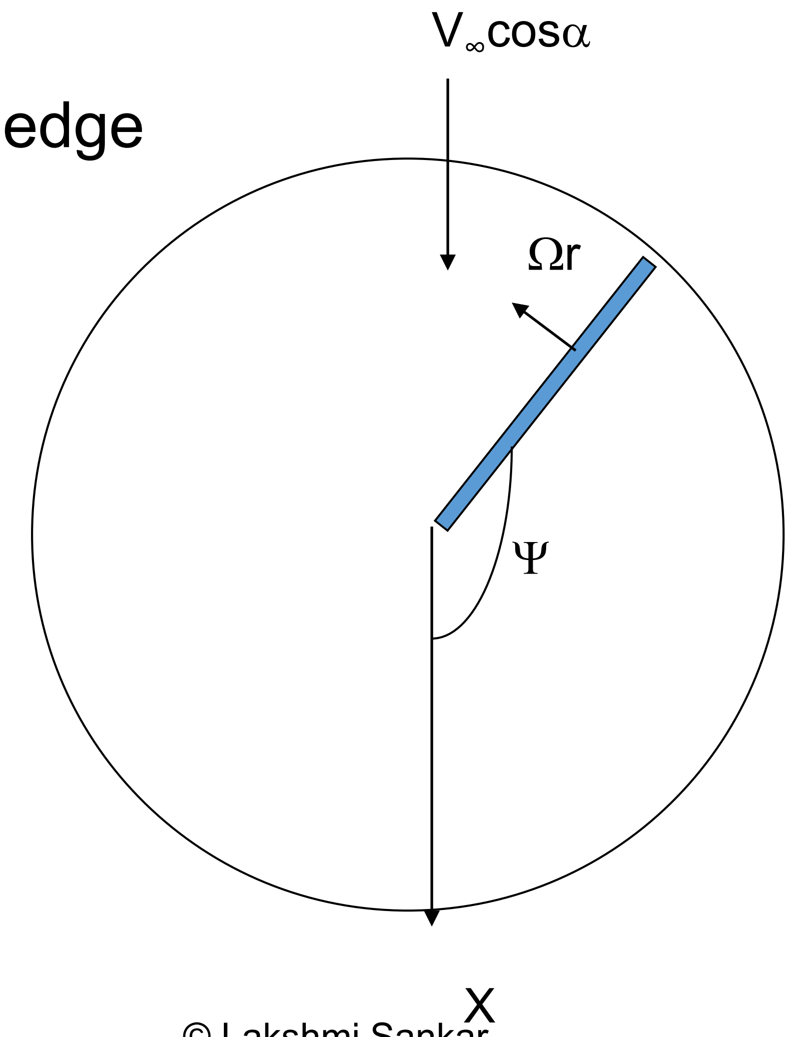
Schematic of Forces and Moments



We assume that the rotor is hinged at the root, for simplicity.
This assumption is adequate for most aerodynamic calculations.
Effects of hinge offset are discussed in many classical texts.

Velocity encountered by the Blade

Velocity normal to
The blade leading edge
is
$\Omega r + V_\infty \sin\Psi \cos\alpha$



Moment at the Hinge due to Aerodynamic Forces

From blade element theory, the lift force dL =

$$\frac{1}{2}\rho c\left[\left(\Omega r + V_\infty \cos\alpha \sin\psi\right)^2\right] C_l dr$$

Moment arm = r cosβ ~ r

Counterclockwise moment due to lift = $\frac{1}{2}\rho c\left(\Omega r + V_\infty \cos\alpha \sin\psi\right)^2 r C_l dr$

Integrating over all such strips,
Total counterclockwise moment = $\int_{r=0}^{r=R} \frac{1}{2}\rho c\left(\Omega r + V_\infty \cos\alpha \sin\psi\right)^2 r C_l dr$

Moment due to Centrifugal Forces

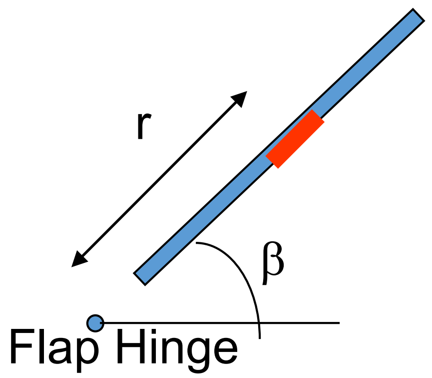
The centrifugal force acting on this strip = $\dfrac{(\Omega r)^2 \, dm}{r} = \Omega^2 r dm$

Where "dm" is the mass of this strip.
This force acts horizontally.
The moment arm = r sinβ ~ r β
Clockwise moment due to centrifugal forces = $\Omega^2 r^2 \beta dm$

Integrating over all such strips, total clockwise moment =

$$\int_{r=0}^{r=R} \Omega^2 r^2 \beta dm \equiv I\Omega^2 \beta$$

Moment at the hinge due to Inertial forces



Small segment of mass dm
With acceleration $r\ddot{\beta}$

β is positive if blade is flapping up

Associated moment at the hinge = $r\left(r\ddot{\beta}\right)dm$

Integrate over all such segments:
Resulting clockwise moment at the hinge= $I\ddot{\beta}$

At equilibrium..

$$I\ddot{\beta} + I\Omega^2\beta = \int_{Root}^{Tip} \frac{1}{2}\rho c C_l \left(\Omega r + V_\infty \cos\alpha \sin\psi\right)^2 r dr$$

Note that the left hand side of this ODE resembles a spring-mass system, with a natural frequency of Ω.

We will later see that the right hand side forcing term has first harmonic (terms containing Ωt), second, and higher Harmonic content.

The system is thus in resonance. Fortunately, there is Adequate aerodynamic damping.

Flapping Dynamics

$$I\ddot{\beta} + I\Omega^2\beta = \int_{Root}^{Tip} \frac{1}{2}\rho c C_l \left(\Omega r + V_\infty \cos\alpha \sin\psi\right)^2 r dr$$

$$\cong \int_{Root}^{Tip} \frac{1}{2}\rho c C_l \left(\Omega r + V_\infty \sin\psi\right)^2 r dr$$

$$C_l = a\alpha_{effective} = a\frac{U_T\theta - U_P}{U_T} = \frac{a}{U_T}\begin{bmatrix} \Omega r\left\{\theta_0 + \left(\theta_{1c} - \beta_{1s}\right)\cos\psi + \left(\theta_{1s} + \beta_{1c}\right)\sin\psi\right\} \\ V_\infty\theta_0 \sin\psi + V_\infty\left(\theta_{1c} - \beta_{1s}\right)\cos\psi \sin\psi \\ +V_\infty\left(\theta_{1s} + \beta_{1c}\right)\sin^2\psi - V_\infty\beta_0 \cos\psi \\ -V_\infty\alpha_{TPP} - \mathrm{v} \end{bmatrix}$$

Solution Process

Assume the blade flapping dynamics equation

$$I\ddot{\beta} + I\Omega^2\beta = \int_{Root}^{Tip} \frac{1}{2}\rho c C_l \left(\Omega r + V_\infty \sin\psi\right)^2 r dr$$

has the general solution of the form

$$\beta = \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi + \beta_{2c}\cos 2\psi + \beta_{2s}\sin 2\psi + \bullet\bullet$$

Plug in the solution on both the left and right sides. The right side Can be integrated analytically, subject to usual assumptions.

Equate coefficients on the left side and right, term by term.
For example coefficient with sinψ on the left with the similar term on right.

Final Form

$$\beta_0 = \gamma\left[\frac{\theta_{80\%R}}{8}\left(1+\mu^2\right) - \frac{\mu^2}{60}\theta_{tw} - \frac{\lambda_{TPP}}{6} + \mu\frac{\beta_{1c}+\theta_{1s}}{6}\right]$$

$$\beta_{1c} + \theta_{1s} = \frac{-\frac{8}{3}\mu\left[\theta_{75\%R} - \frac{3}{4}\lambda_{TPP}\right]}{1+\frac{3}{2}\mu^2}$$

$$\beta_{1s} - \theta_{1c} = \frac{-\frac{4}{3}\mu\beta_0}{1+\frac{1}{2}\mu^2}$$

$$C_T = \frac{\sigma a}{2}\left[\frac{\theta_0}{3}\left\{1+\frac{3}{2}\mu^2\right\} + \frac{\theta_{tw}}{4}\left\{1+\mu^2\right\} + \mu\frac{\theta_{1s}+\beta_{1c}}{2} - \frac{\lambda_{TPP}}{2}\right]$$

Inverting the above system..
From Wayne Johnson's Text

$$\theta_{.75} = \frac{\left(1+\frac{3}{2}\mu^2\right)\left(\frac{6C_T}{\sigma a}+\frac{3}{8}\mu^2\theta_{tw}\right)+\frac{3}{2}\lambda_{TPP}\left(1-\frac{1}{2}\mu^2\right)}{1-\mu^2+\frac{9}{4}\mu^4}$$

$$\theta_{1s} = -\beta_{1c} - \frac{\frac{8}{3}\mu\left(\frac{6C_T}{\sigma a}+\frac{3}{8}\mu^2\theta_{tw}\right)+2\mu\lambda_{TPP}\left(1-\frac{3}{2}\mu^2\right)}{1-\mu^2+\frac{9}{4}\mu^4}$$

$$\beta_0 = \frac{\gamma/8}{1-\mu^2+\frac{9}{4}\mu^4}\left[\left(1-\frac{19}{18}\mu^2+\frac{3}{2}\mu^4\right)\frac{6C_T}{\sigma a} + \left(\frac{1}{20}+\frac{29}{120}\mu^2-\frac{1}{5}\mu^4+\frac{3}{8}\mu^6\right)\theta_{tw} + \left(\frac{1}{6}-\frac{7}{12}\mu^2+\frac{1}{4}\mu^4\right)\lambda_{TPP}\right]$$

$$\theta_{1c} = \beta_{1s} + \frac{\frac{4}{3}\mu\beta_0}{1+\frac{1}{2}\mu^2}$$