Forward Flight

Integration of Sectional Loads

To get

Total Loads at the Hub

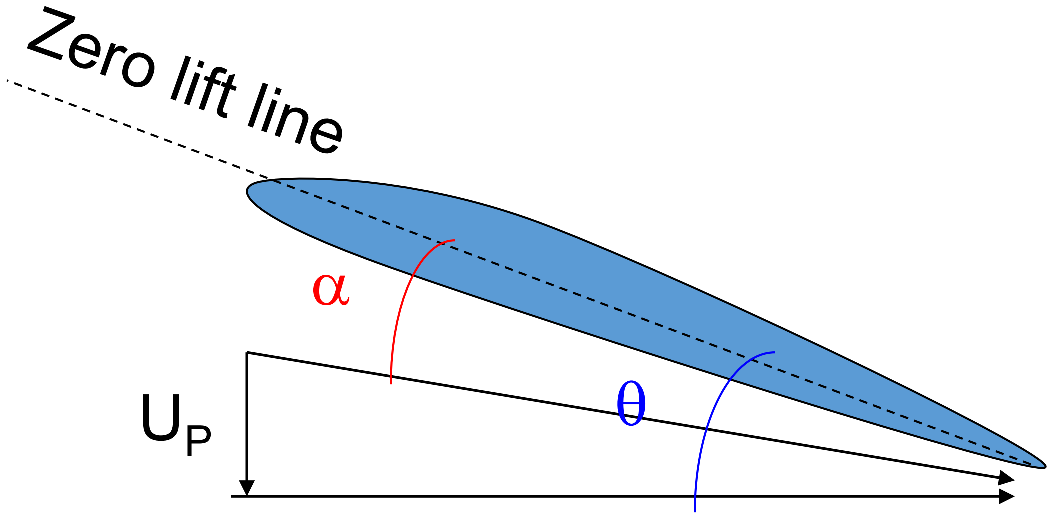
Background

- In the previous sections, we discussed how to compute the angle of attack of a typical blade element.
- We also discussed how to compute lift, drag, and pitching moment coefficients.
- We also discussed how to compute sectional lift and drag forces per unit span.
- We mentioned that these loads must be rotated to get components normal to, and along reference plane.
- In this section, we discuss how to integrate these loads.
- In computer codes, these integrations are done numerically.
- Analytical integration under simplifying assumptions will be given here to illustrate the process.

Assumptions for Analytical Integration

- c= constant (untapered rotor)
- v = constant (uniform inflow)
- Cd = constant
- Linearly twisted rotor
- No cut out, no tip losses.

Blade Section



$U_T = \Omega r + V_\infty \cos\alpha \sin\psi \approx \Omega r + V_\infty \sin\psi$

Effective Angle of Attack

As discussed earlier, $\alpha_{eff} = \theta - \frac{U_P}{U_T}$

$$U_T = \Omega r + V_\infty \sin\psi$$

$$\theta = \theta_{root} + \theta_{twist}\frac{r}{R} = \theta_0 + \theta_{twist}\frac{r}{R}$$

$$\begin{aligned} U_P &= V_\infty \sin\alpha_{TPP} + \mathrm{v} + V_\infty \beta\cos\psi \\ &= \Omega R \lambda_{TPP} + V_\infty \beta \cos\psi \end{aligned}$$

$$L' = \frac{1}{2}\rho c C_l\left(U_T^2 + U_P^2\right) \approx \frac{1}{2}\rho c a U_T^2 \alpha_{eff} = \frac{1}{2}\rho c a\left(\theta U_T^2 - U_T U_P\right)$$

Some algebra first..

$$L' \equiv \frac{1}{2}\rho c a\left[\begin{array}{l}(\Omega r + V_\infty \sin\psi)^2\left(\theta_0 + \theta_{tw}\frac{r}{R} + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi\right) \\ -(\Omega r + V_\infty \sin\psi)(\Omega R\lambda_{TPP} + V_\infty \beta\cos\psi)\end{array}\right]$$

$$= \frac{1}{2}\rho c a\left[\begin{array}{l}\theta_0\Omega^2 r^2 + 2V_\infty\Omega r\theta_0\sin\psi + \theta_0 V_\infty^2\sin^2\psi + \theta_{tw}\Omega^2\frac{r^3}{R} + 2V_\infty\Omega\theta_{tw}\frac{r^2}{R}\sin\psi + \\ \theta_{tw}\frac{r}{R}V_\infty^2\sin^2\psi + \Omega^2 r^2\theta_{1c}\cos\psi + 2\Omega r V_\infty\theta_{1c}\sin\psi\cos\psi + \\ V_\infty^2\theta_{1c}\sin^2\psi\cos\psi + \\ \Omega^2 r^2\theta_{1s}\sin\psi + 2\Omega r V_\infty\theta_{1s}\sin^2\psi + V_\infty^2\theta_{1s}\sin^3\psi \\ -\Omega^2 R\lambda_{TPP} r - V_\infty\Omega R\lambda_{TPP}\sin\psi - \Omega r V_\infty\beta\cos\psi - V_\infty^2\beta\cos\psi\sin\psi\end{array}\right]$$

Notice that we have first, second, and third harmonics present! These fluctuations will be felt by the passengers/pilots as vibratory loads.

Thrust

$$T = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left(\int_0^{R} L' dr\right) d\psi\right]$$

Thrust is computed by integrating the lift radially to get
instantaneous thrust force at the hub,
then averaging the thrust force over the entire rotor disk,
and multiplying the force per blade by the number of blades.

Computer codes will do the integrations numerically, without any of the assumptions we had to make.

Anlaytical Integration of Thrust

We can interchange the order of integration.

Integrate with respect to ψ first. Use the formulas such as

$$\int_0^{2\pi} \sin\psi d\psi = \int_0^{2\pi} \cos\psi d\psi = \int_0^{2\pi} \sin^2\psi \cos\psi d\psi = 0$$

$$\int_0^{2\pi} \sin\psi \cos\psi \mathrm{d}\psi = 0$$

$$\int_0^{2\pi} \sin^2\psi d\psi = \int_0^{2\pi} \cos^2\psi d\psi = \pi$$

Result of Azimuthal Integration

$$\frac{1}{2\pi}\int_0^{2\pi} L' d\psi = \frac{1}{2}\rho c a \left[\begin{array}{l} \theta_0 \Omega^2 r^2 + \frac{\theta_0}{2} V_\infty^2 \\ + \theta_{tw} \Omega^2 \frac{r^3}{R} + V_\infty^2 \theta_{tw} \frac{r}{R} \\ + \Omega r V_\infty \theta_{1s} - \Omega^2 R r \lambda_{TPP} \end{array} \right]$$

Next perform radial integration and Normalize

$$T = \frac{1}{2}\rho abc\left[\begin{array}{l}\theta_0\Omega^2\dfrac{R^3}{3}+\dfrac{\theta_0}{2}V_\infty^2 R+\theta_{tw}\Omega^2\dfrac{R^3}{4}+\dfrac{\theta_{tw}}{4}V_\infty^2 R+\\ \Omega V_\infty \theta_{1s}\Big|_{TPP}\dfrac{R^2}{2}-\Omega^2\lambda_{TPP}\dfrac{R^2}{2}\end{array}\right]$$

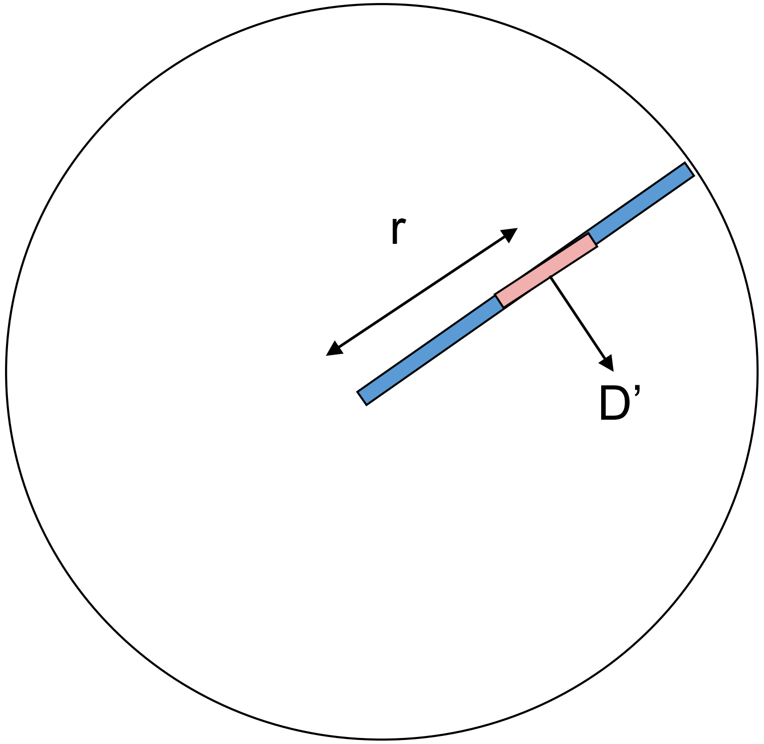
$$C_T = \frac{\sigma a}{2}\left[\frac{\theta_0}{3}\left\{1+\frac{3}{2}\mu^2\right\}+\frac{\theta_{tw}}{4}\left\{1+\mu^2\right\}+\mu\frac{\theta_{1s}\big|_{TPP}}{2}-\frac{\lambda_{TPP}}{2}\right]$$

Note that we will get the hover expressions back if advance ratio μ is set to zero.

Torque and Power

- We next look at how to compute the instantaneous torque and power on a blade.
- These are azimuthally-averaged to get total torque and total power.
- It is simpler to look at profile and induced components of torque are power separately.

Profile Drag



$$D' = \frac{1}{2}\rho U_T^2 c C_{d,0}$$

where

$$U_T = \Omega r + V_\infty \sin\psi$$

We will assume chord c and drag coefficient C_{d0} are constant.

Integration of Profile Torque

$$Q_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R rD'dr\right\}\right]$$

Non-dimensionalize: $$C_{Q,0} = \frac{Q}{\rho(\Omega R)^2 AR}$$

Final result: $$C_{Q,0} = \frac{\sigma C_{d,0}}{8}\left[1+\mu^2\right]$$

Profile Power

$$Q_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R rD'dr\right\}\right]$$

$$P_0 = b\left[\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_0^R U_T D'dr\right\}\right]$$

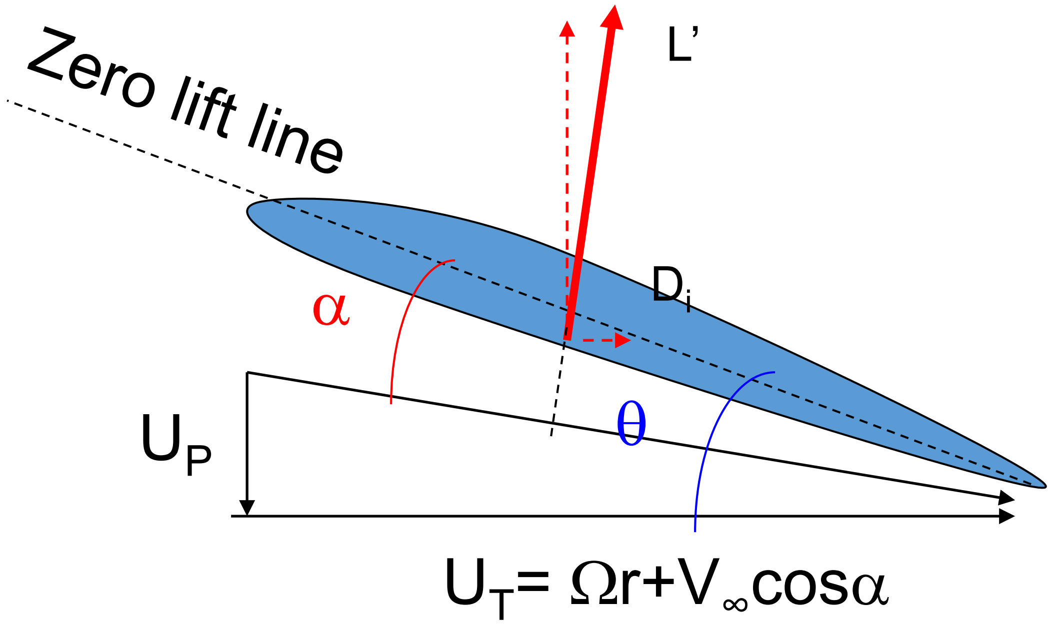
Non-dimensionalize:

$$C_{P,0} = \frac{P_0}{\rho(\Omega R)^3 A}$$

Final result:

$$C_{P,0} = \frac{\sigma C_{d,0}}{8}\left[1+3\mu^2\right]$$

Induced Drag



$$D_i' \cong L' \frac{U_P}{U_T} = \frac{1}{2} \rho c C_l U_T^2 \frac{U_P}{U_T}$$

$$= \frac{1}{2} \rho c C_l U_T U_P$$

Induced Torque and Power

$$Q_i = \frac{1}{2\pi} \int_0^{2\pi} \left[\int_0^R r D_i dr \right] d\psi$$

$$P_i = \frac{1}{2\pi} \int_0^{2\pi} \left[\int_0^R U_T D_i dr \right] d\psi$$

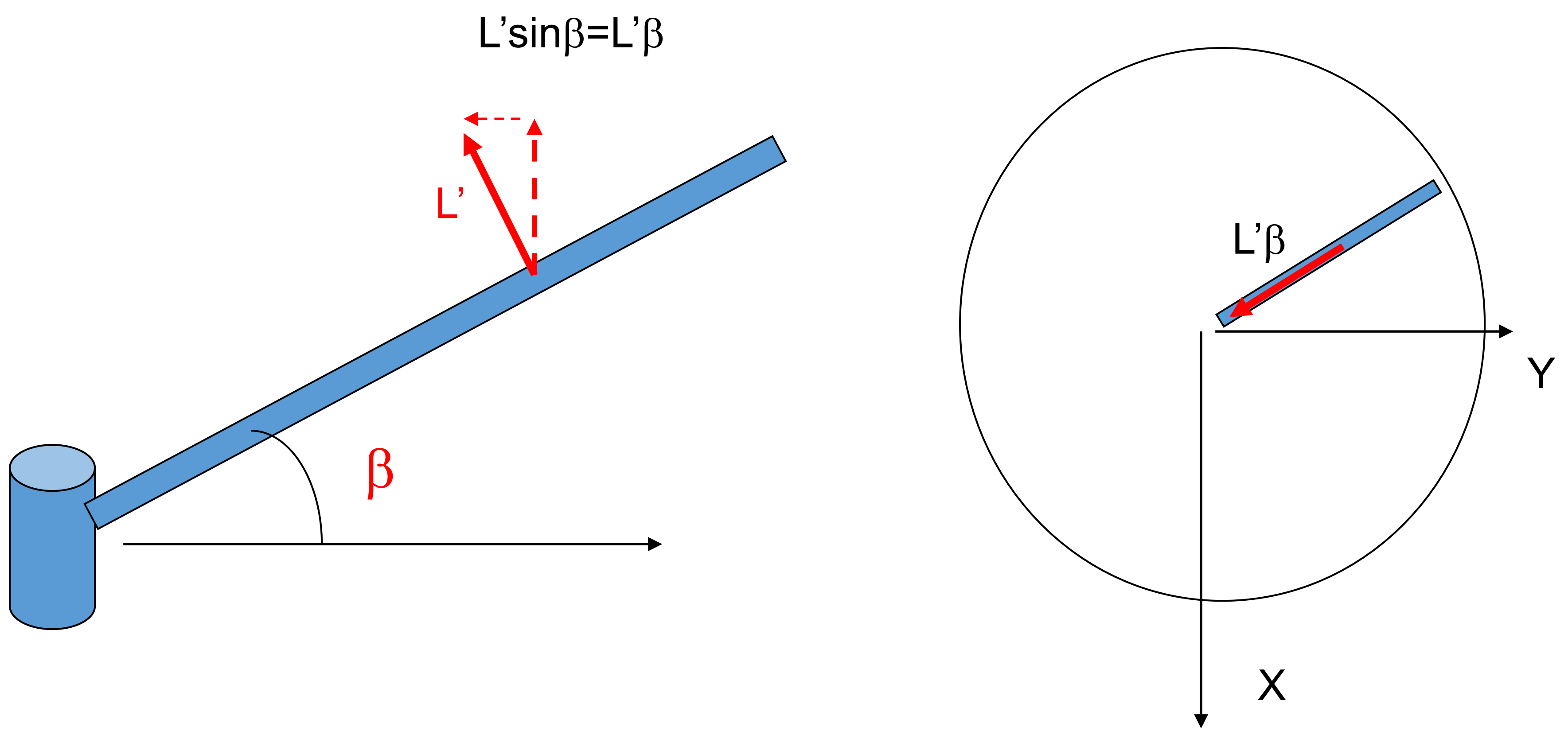
Performing the analytical integration, $$C_{P_i} = \lambda_i C_T$$

This is a familiar result. Induced Power = Thrust times Induced Velocity!

In-Plane Forces

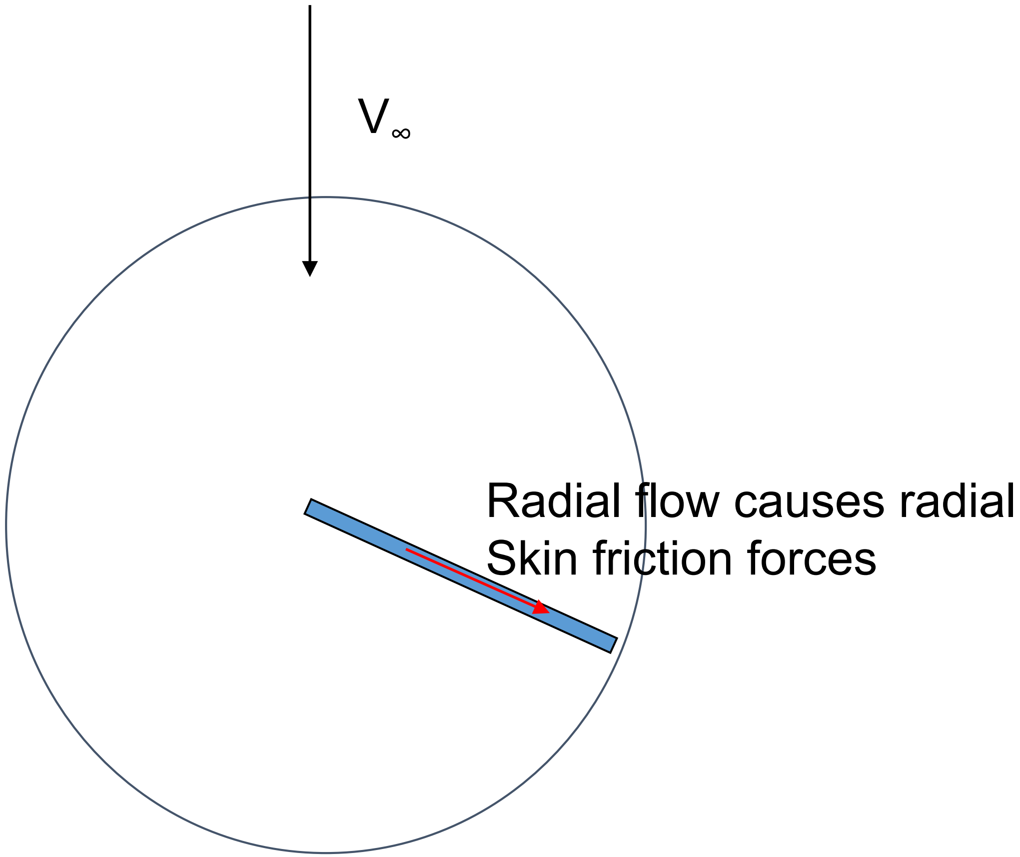
- In addition to thrust, that act normal to the rotor disk (or along the z-axis in the coordinate selected by the user), the blade sections generate in-plane forces.
- These forces must be integrated to get net force along the x- axis. This is called the H-force.
- These forces must be integrated to get net forces along the Y- axis. This is called the Y-force.
- These forces will have inviscid components, and viscous components.

Origin of In-Plane Forces



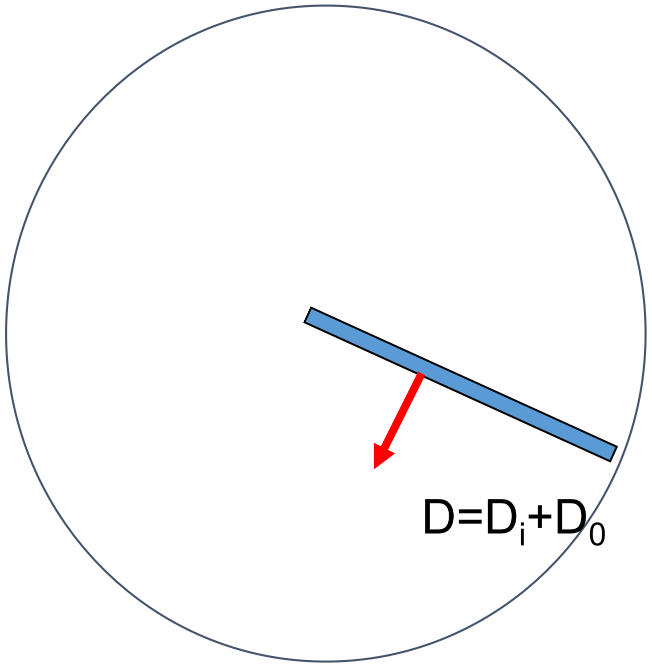
One source of in-plane forces is the tilting of the Thrust due to the blade coning angle.

Origin of In-Plane Forces-II



A component of the free-stream flows along the blade,
in the radial direction. This causes radial skin friction forces.
This is hard to quantify, and is usually neglected.

Origin of In-Plane Forces III



Sectional drag (which is made of inviscid induced drag, and viscous drag)
Can give rise to components along the X- direction (H-force), and
Y- direction (Y-Force).
Engineers are interested in both the instantaneous values (which determine
Vibration levels), as well as azimuthal averages (which determine force balance).

Closed Form Expressions for C_H and C_Y

- Under our assumptions of constant chord, linear twist, linear aerodynamics, and uniform inflow, these forces may be integrated radially, and averaged azimuthally.
- The H- forces and the Y- forces are non-dimensionalized the same way thrust is non-dimensionalized.
- Many text books (e.g. Leishman, Prouty) give exact expressions for these coefficients.

Closed Form Expressions

$$C_{H_{TPP}} = \mu \frac{\sigma C_{d0}}{4} + \frac{\sigma a}{2} \left[\begin{array}{l} \frac{\mu \lambda_{TPP}}{2}\left(\theta_0 + \frac{\theta_{tw}}{2}\right) - \\ \frac{\theta_{1c}\beta_0}{6} + \theta_{1s}\frac{\lambda_{TPP}}{4} \\ + \frac{\mu \beta_0^2}{4} \end{array} \right]$$

$$C_{Y_{TPP}} = \frac{\sigma a}{2} \left[\begin{array}{l} \frac{3}{4}\mu\beta_0\left(\theta_0 + \frac{2}{3}\theta_{tw}\right) + \frac{\theta_{1c}}{4}\lambda_{TPP} \\ + \frac{\theta_{1s}\beta_0}{6}\left(1 + 3\mu^2\right) - \frac{3}{2}\mu\beta_0\lambda_{TPP} \end{array} \right]$$