Hover Performance Prediction Methods

Combined Blade Element-Momentum (BEM) Theory

Drawbacks of Blade Element Theory

- It does not handle tip losses.
- It assumes that the induced velocity v is uniform.
- It does not account for swirl losses.
- The Predicted power is sometimes empirically corrected for these losses.

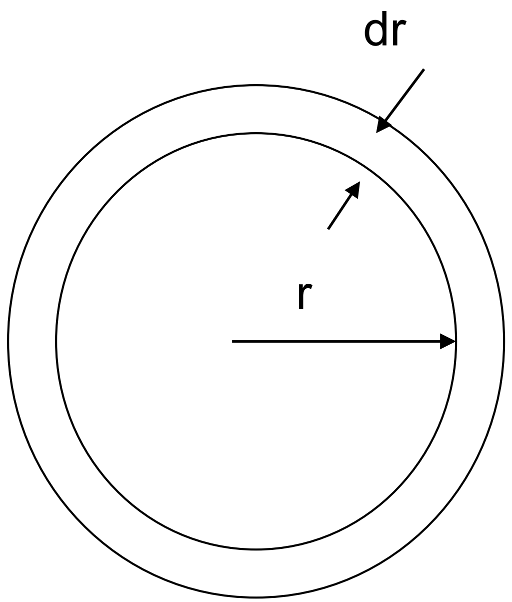
$$C_P = \kappa \lambda C_T + \frac{\sigma C_{d0}}{8}$$

$$\kappa = 1.15$$

Background

- Blade Element Theory has a number of assumptions.
- The biggest (and worst) assumption is that the inflow is uniform.
  - 9% under-prediction of induced power results if we assume that the inflow is uniform
- In reality, the inflow is non-uniform.
  - It may be shown from variational calculus that uniform inflow yields the lowest induced power consumption.

Consider an Annulus of the rotor Disk



Area = $2\pi r dr$

Mass flow rate $= 2\pi r \rho (V+v) dr$

dT = (Mass flow rate) * (twice the induced velocity at the annulus)

$= 4\pi\rho r(V+v)v dr$

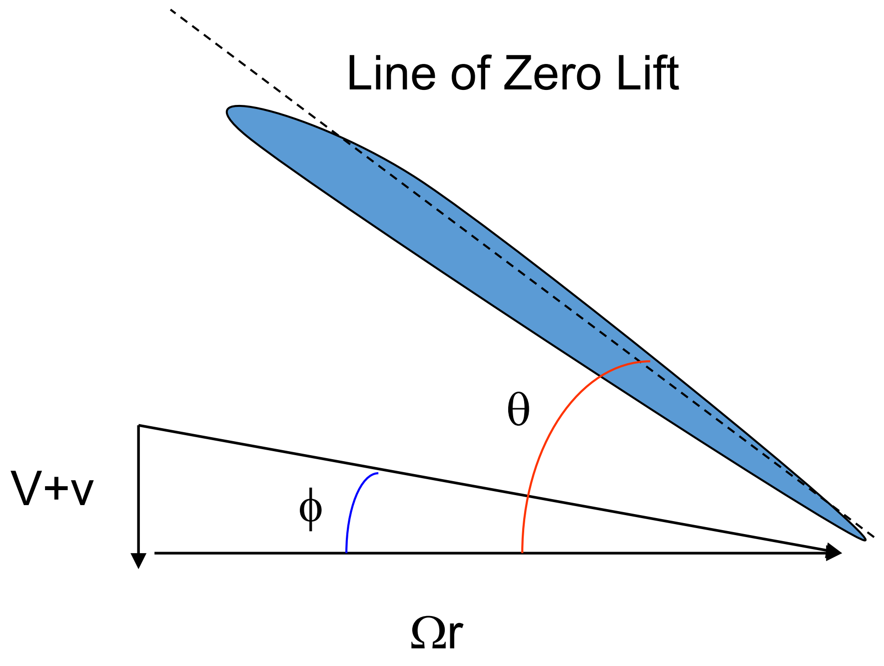
Typical Airfoil Section

$$\phi = \arctan\left(\frac{V + \mathrm{v}}{\Omega r}\right)$$

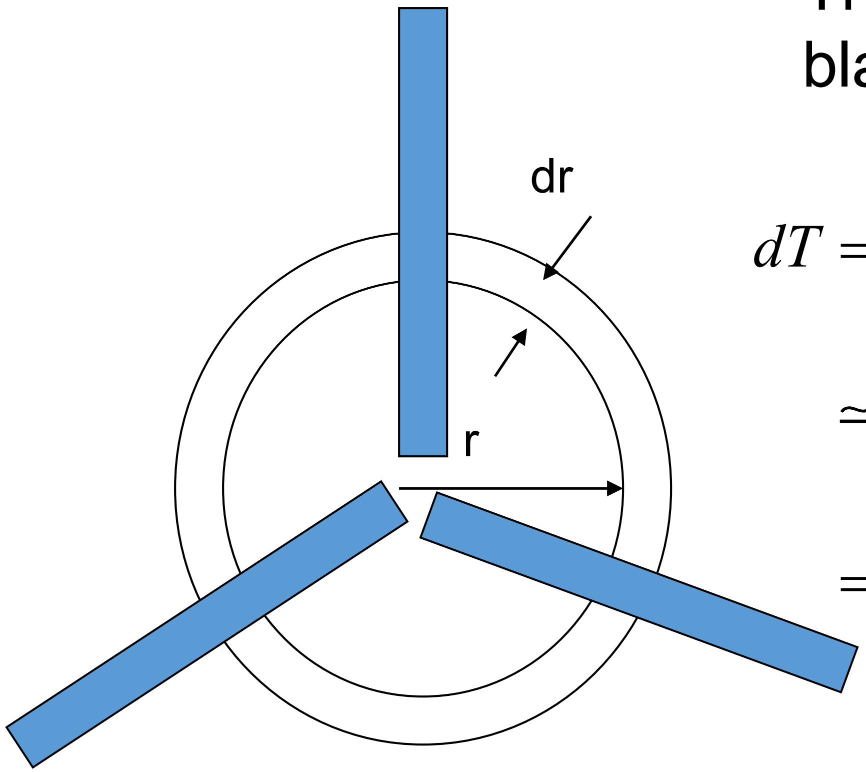
$\alpha = \theta - \phi$

$C_\ell = a\,(\theta - \phi)$

a : Lift curve slope



Blade Elements Captured by the Annulus



Thrust generated by these blade elements:

$$dT = b \cdot \frac{1}{2}\rho \cdot \left[\left(\Omega r\right)^2 + \left(V + \mathrm{v}\right)^2 \right] c \cdot C_l \cdot dr$$

$$\simeq b \cdot \frac{1}{2}\rho \cdot \left[\left(\Omega r\right)^2 \right] c \cdot C_l \cdot dr$$

$$= abc \cdot \frac{1}{2}\rho \cdot \left(\Omega r\right)^2 \cdot \left(\theta - \frac{V + \mathrm{v}}{\Omega r} \right) \cdot dr$$

Equate the Thrust for the Elements from the Momentum and Blade Element Approaches

$$\lambda^2 + \left(\frac{\sigma a}{8} - \lambda_c\right)\lambda - \frac{\sigma a}{8}\theta\frac{r}{R} = 0$$

where,

$$\lambda_c = \frac{V}{\Omega R}$$

$$\lambda = \frac{V + \mathrm{v}}{\Omega \mathrm{R}}$$

$$\lambda = \sqrt{\left(\frac{\sigma a}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma a}{8}\theta\frac{r}{R}} - \left(\frac{\sigma a}{16} - \frac{\lambda_c}{2}\right)$$

Total Inflow Velocity from Combined Blade Element-Momentum Theory

Numerical Implementation of Combined BEM Theory

- The numerical implementation is identical to classical blade element theory.
- The only difference is the inflow is no longer uniform. It is computed using the formula given earlier, reproduced below:

$$\lambda = \sqrt{\left(\frac{\sigma a}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma a}{8}\theta\frac{r}{R}} - \left(\frac{\sigma a}{16} - \frac{\lambda_c}{2}\right)$$

Note that inflow is uniform if θ= CR/r . This twist is therefore called the ideal twist.

Effect of Inflow on Power in Hover

$$P_{induced} = \int_0^R \mathrm{v}\,dT = \int_0^R 4\rho\pi r \mathrm{v}^3 dr$$

$$T = \int_0^R dT = \int_0^R 4\rho\pi r \mathrm{v}^2 dr$$ ← constraint

We wish to minimize induced power, for a specified value of T.

Therefore, we minimize P - βT where β is a Lagrangean multiplier. $\delta(P - \beta T) = 0$

↑ Variation of a functional

$$\delta\left[\int_0^R \left[4\rho\pi r \mathrm{v}^3 - 4\beta\rho\pi r \mathrm{v}^2\right] dr\right] = 0$$

$$\int_0^R 4\rho\pi r\left[3\mathrm{v}^2 - 2\beta \mathrm{v}\right]\delta \mathrm{v}\, dr = 0$$

The only way the integral will vanish for all possible variations δv is if $\left[3\mathrm{v}^2 - 2\beta \mathrm{v}\right] = 0$

Since β is a contant (Lagrangean multiplier), it follows that v must be a constant.

Uniform inflow produces least induced power, for a specified level of thrust!

Ideal Rotor vs. Optimum Rotor

- Ideal rotor has a non-linear twist: $\theta = CR/r$
- This rotor will, according to the BEM theory, have a uniform inflow, and the lowest induced power possible.
- The rotor blade will have very high local pitch angles θ near the root, which may cause the rotor to stall.
- Ideally Twisted rotor is also hard to manufacture.
- For these reasons, helicopter designers strive for optimum rotors that minimize total power, and maximize figure of merit.
- This is done by a combination of twist, and taper, and the use of low drag airfoil sections.

Optimum Rotor

- We try to minimize total power (Induced power + Profile Power) for a given T.
- In other words, an optimum rotor has the maximum figure of merit.
- From earlier work (see slide 72), figure of merit is maximized if $C_l^{3/2} / C_d$ is maximized.
- All the sections of the rotor will operate at the angle of attack where this value of C_l and C_d are produced.
- We will call this C_l the optimum lift coefficient $C_{l,optimum}$.

Optimum rotor (continued..)

All radial stations will operate at an optimum a at which $\frac{C_l^{3/2}}{C_d}$ is maximum.

Once angle of attack α is selected, we find θ from

$$\alpha = \theta - \arctan\left(\frac{v}{\Omega r}\right) \text{ and } \frac{v}{\Omega R} = \sqrt{\frac{C_T}{2}}$$

This determines how the blade must be twisted.

Variation of Chord for the Optimum Rotor

$$dT = b \cdot \frac{1}{2}\rho \cdot (\Omega r)^2 c \cdot C_l \cdot dr$$

dT = (Mass flow rate) * (twice the induced velocity at the annulus)
= $4\pi\rho r(v)vdr$

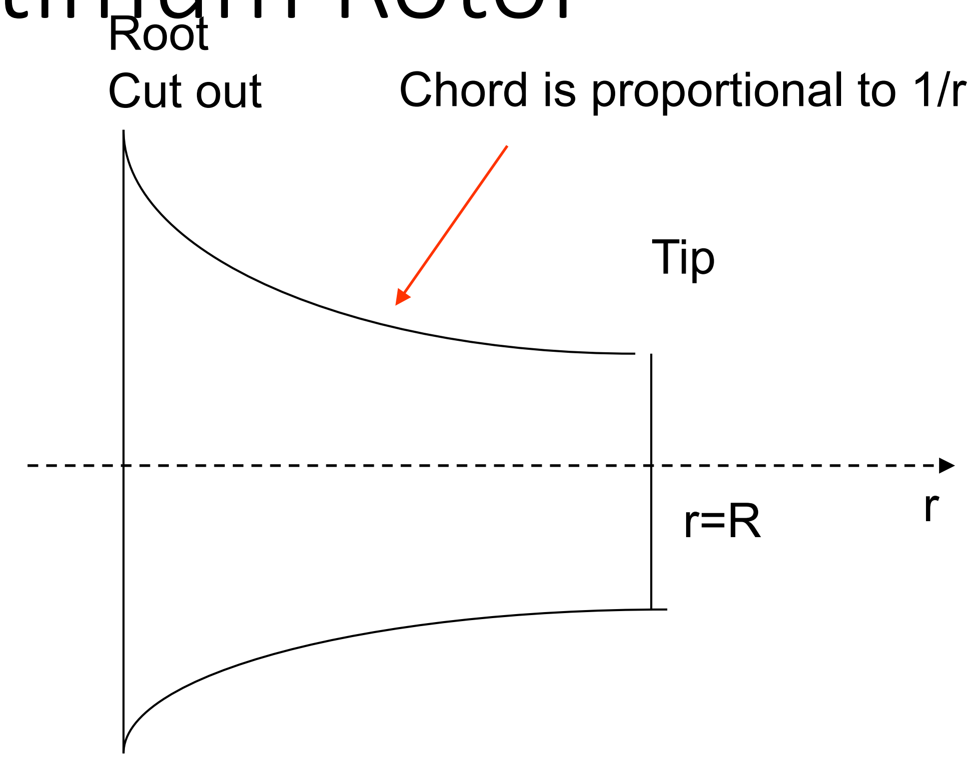
Compare these two. Note that C_l is a constant (the optimum value).

It follows that

$$\sigma(r) = \frac{bc}{\pi R} = \left(\frac{8v^2}{\Omega^2 R C_l}\right)\frac{1}{r} = \frac{Const}{r}$$

↑ Local solidity

Planform of Optimum Rotor



Such planforms and twist distributions are hard to manufacture, and are optimum only at one thrust setting.

Manufacturers therefore use a combination of linear twist, and linear variation in chord (constant taper ratio) to achieve optimum performance.

Accounting for Tip Losses

- We have already accounted for two sources of performance loss-non-uniform inflow, and blade viscous drag.
- We can account for compressibility wave drag effects and associated losses, during the table look-up of drag coefficient.
- Two more sources of loss in performance are tip losses, and swirl.
- An elegant theory is available for tip losses from Prandtl.

Prandtl's Tip Loss Model

Prandtl suggests that we multiply the sectional inflow by a function F, which goes to zero at the tip, and unity in the interior.

$$F = \frac{2}{\pi} arcCos\left(e^{-f}\right)$$

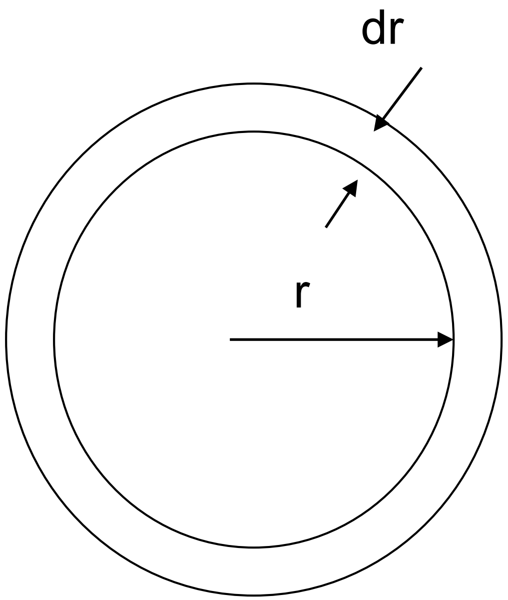
$$where,$$

When there are infinite number of blades, F approaches unity, there is no tip loss.

$$f = \frac{b}{2}\frac{\left(1 - r / R\right)}{\lambda}$$

Incorporation of Tip Loss Model in BEM

All we need to do is multiply the lift due to inflow by F.



Thrust generated by the annulus:

$$dT = $$
$$= 4\pi\rho r F (V+v) v dr$$

Resulting Inflow (Hover)

$$\lambda = \sqrt{\left(\frac{\sigma a}{16F}\right)^2 + \frac{\sigma a}{8F}\theta\frac{r}{R}} - \left(\frac{\sigma a}{16F}\right)$$

$$= \frac{\sigma a}{16F}\left[\sqrt{1+\frac{32F}{\sigma a}\theta\frac{r}{R}} - 1\right]$$