Hover Performance Prediction Methods

II. Blade Element Theory

Preliminary Remarks

- Momentum theory gives rapid, back-of-the-envelope estimates of Power.
- This approach is sufficient to size a rotor (i.e. select the disk area) for a given power plant (engine), and a given gross weight.
- This approach is not <u>adequate</u> for designing the rotor.

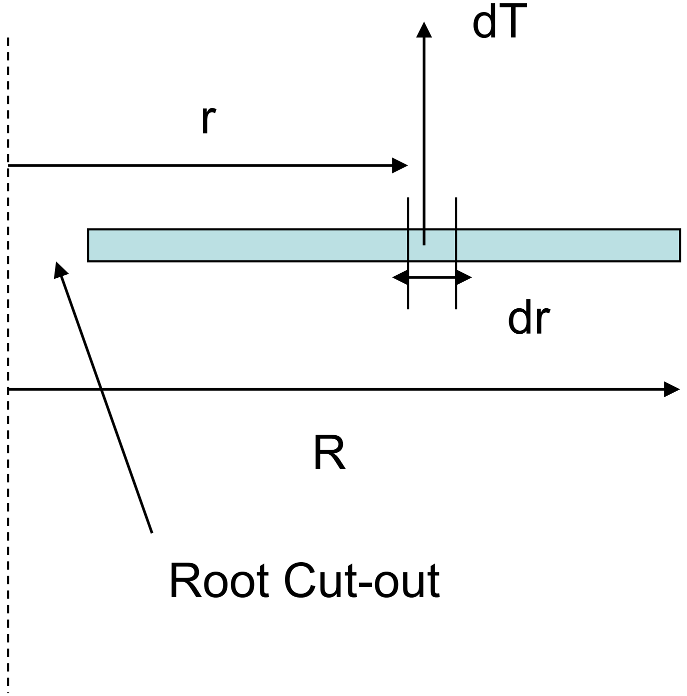
Drawbacks of Momentum Theory

- It does not take into account
  - Number of blades
  - Airfoil characteristics (lift, drag, angle of zero lift)
  - Blade planform (taper, sweep, root cut-out)
  - Blade twist distribution
  - Compressibility effects

Blade Element Theory

- Blade Element Theory rectifies many of these drawbacks. First proposed by Drzwiecki in 1892.
- It is a “strip” theory. The blade is divided into a number of strips, of width Δr.
- The lift generated by that strip, and the power consumed by that strip, are computed using 2-D airfoil aerodynamics.
- The contributions from all the strips from all the blades are summed up to get total thrust, and total power.

Typical Blade Section (Strip)

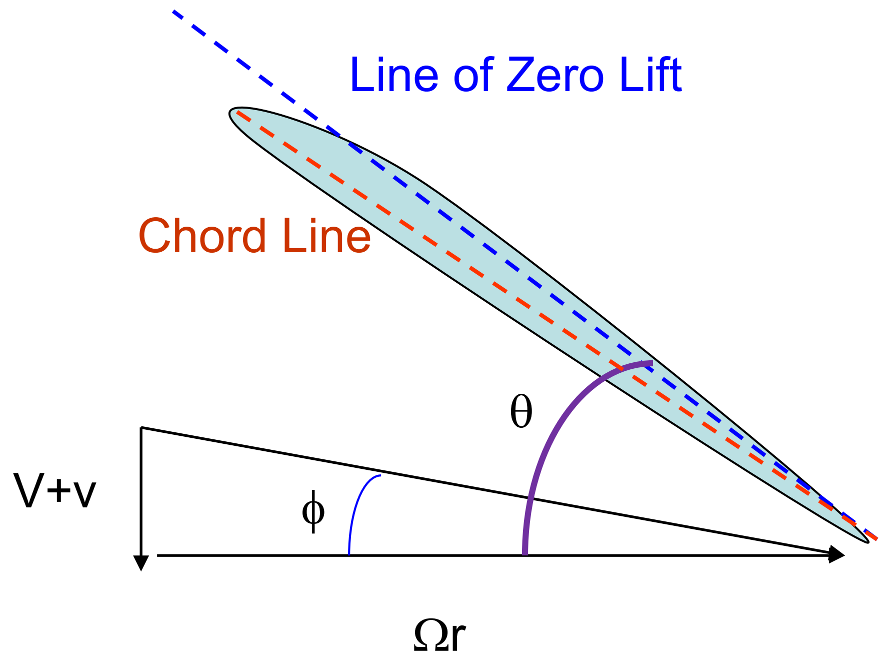


$$T = b \int_{Cut-Out}^{Tip} dT$$

$$P = b \int_{Cut-Out}^{Tip} dP$$

Typical Airfoil Section

$$\phi = \arctan\left(\frac{V + \mathrm{v}}{\Omega r}\right)$$



$$\alpha_{\text{effective}} = \theta - \phi$$

Sectional Forces

Once the effective angle of attack is known, we can look-up the lift and drag coefficients for the airfoil section at that strip.

We can subsequently compute sectional lift and drag forces per foot (or meter) of span.

$$\Delta L = \frac{1}{2}\rho\left(U_T^2 + U_P^2\right)cC_l$$

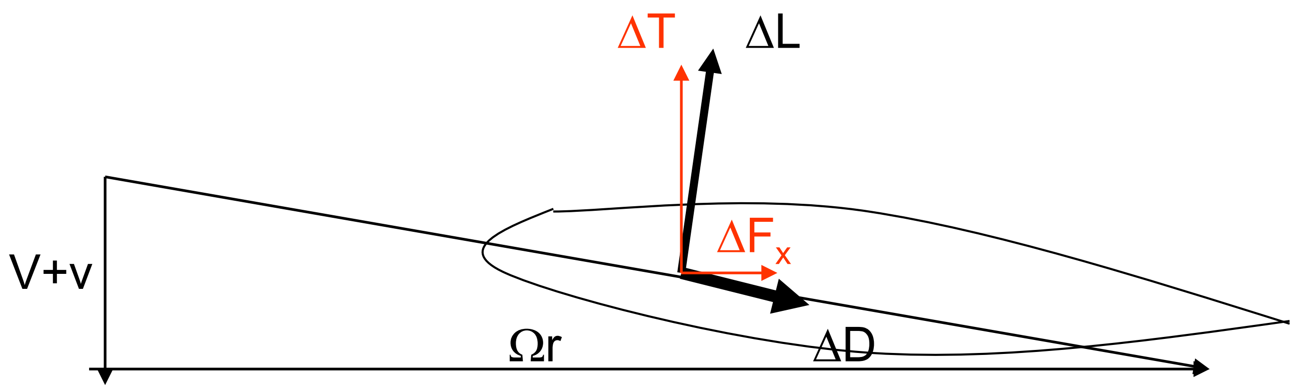
$$\Delta D = \frac{1}{2}\rho\left(U_T^2 + U_P^2\right)cC_d$$

$U_T=\omega r$

$U_P=V+v$

These forces will be normal to and along the total velocity vector.

Rotation of Forces



$$dT = \left(\Delta L \cos(\phi) - \Delta D \sin(\phi)\right) dr$$

$$= \frac{1}{2}\rho\left(U_T^2 + U_P^2\right)c\left(C_l \cos(\phi) - C_d \sin(\phi)\right) dr$$

$$dF_x = \left(\Delta D \cos(\phi) + \Delta L \sin(\phi)\right) dr$$

$$= \frac{1}{2}\rho\left(U_T^2 + U_P^2\right)c\left(C_d \cos(\phi) + C_l \sin(\phi)\right) dr$$

$$dP = U_T dF_x = \Omega r dF_X$$

Approximate Expressions

- The integration (or summation of forces) can only be done numerically.
- A spreadsheet may be designed. A sample spreadsheet is being provided as part of the course notes.
- In some simple cases, analytical expressions may be obtained.

Closed Form Integrations

- The chord c is constant. Simple linear twist.
- The inflow velocity v and climb velocity V are small. Thus, $\phi << 1$.
- We can approximate $\cos(\phi)$ by unity, and approximate $\sin(\phi)$ by (ϕ).
- The lift coefficient is a linear function of the effective angle of attack, that is, $Cl=a(\theta-\phi)$ where a is the lift curve slope.
- For low speeds, a may be set equal to 5.7 per radian.
- C_d is small. So, $C_d \sin(\phi)$ may be neglected.
- The in-plane velocity Ωr is much larger than the normal component V+v over most of the rotor.

Closed Form Expressions

$$T = \frac{1}{2}\rho cba\Omega^2 \int_{r=0}^{r=R} \left(\theta - \frac{V}{\Omega r} - \frac{\mathrm{v}}{\Omega r}\right) r^2 dr$$

$$P = \frac{1}{2}\rho cb$$

$$\Omega^3 \int_{r=0}^{r=R} \left[a\left(\theta - \frac{V}{\Omega r} - \frac{\mathrm{v}}{\Omega r}\right)\left(\frac{V}{\Omega r} + \frac{\mathrm{v}}{\Omega r}\right) + C_d \right] r^3 dr$$

Linearly Twisted Rotor: Thrust

Here, we assume that the pitch angle varies as

$$\theta = E + Fr$$

$$T = \frac{b}{2}\rho\Omega^2 ca\left[\frac{1}{3}\left(E + \frac{3}{4}FR\right) - \frac{V+v}{2\Omega R}\right]R^3 = \frac{b}{2}\rho ca(\Omega R)^2 R\left[\frac{\theta_{.75R}}{3} - \lambda / 2\right]$$

$$C_T = \frac{abc}{2\pi R}\left[\frac{\theta_{.75}}{3} - \lambda / 2\right] = \frac{a\sigma}{2}\left[\frac{\theta_{.75R}}{3} - \lambda / 2\right]$$

where

$$\sigma = \text{solidity} = \text{BladeArea/DiskArea} = bc / \pi R$$

$$a = \text{Lift Curve slope} \ (\sim 2\pi)$$

$$\lambda = \text{Inflow Ratio} = \frac{V+v}{\Omega R}$$

Linearly Twisted Rotor

Notice that the thrust coefficient is linearly proportional to the pitch angle θ at the 75% Radius.

This is why the pitch angle is usually defined at 75% R in industry.

The expression for power may be integrated in a similar manner, if the drag coefficient C_d is assumed to be a constant, equal to C_{d0}.

$$C_P = \underbrace{\lambda C_T}_{\text{Induced Power}} + \underbrace{\frac{\sigma C_{d0}}{8}}_{\text{Profile Power}}$$

Closed Form Expressions for Ideally Twisted Rotor

$$\theta = \frac{\theta_{tip} R}{r}$$

$$C_T = \frac{\sigma a}{4}\left(\theta_{tip} - \lambda\right)$$

$$C_P = \lambda C_T + \frac{\sigma C_{d0}}{8}$$

Same as linearly Twisted rotor!

Figure of Merit according to Blade Element Theory

$$FM = \frac{\lambda C_T}{\lambda C_T + \sigma C_{d0}/8}$$

$$where,$$

$$\lambda = \text{Inflow Ratio} = (\text{V} + \text{v})/\Omega\text{R}$$

$$\sigma = \text{Solidity} = \text{Blade Area/Disk Area}$$

High solidity (lot of blades, wide-chord, large blade area) leads to higher Power consumption, and lower figure of merit.

Figure of merit can be improved with the use of low drag airfoils.

Average Lift Coefficient

- Let us assume that every section of the entire rotor is operating at an optimum lift coefficient.
- Let us assume the rotor is untapered.

$$\text{Average Lift Coefficient} = \overline{C}_l$$

$$T = b\int_0^R \frac{1}{2}\rho c(\Omega r)^2 \overline{C}_l dr = \frac{\rho b c \overline{C}_l \Omega^2 R^3}{6}$$

$$C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} = \frac{bc}{\pi R}\frac{\overline{C}_l}{6} = \sigma \frac{\overline{C}_l}{6}$$

$$\overline{C}_l = 6\frac{C_T}{\sigma}$$

Rotor will stall if average lift coefficient exceeds 1.2, or so.

Thus, in practice, C_T/σ is limited to 0.2 or so.

Optimum Lift Coefficient in Hover

$$FM = \frac{\lambda C_T}{\lambda C_T + \frac{\sigma C_{d0}}{8}}$$

$$\text{In hover, } \lambda = \sqrt{\frac{\mathrm{C_T}}{2}}$$

$$FM = \frac{C_T^{3/2} \big/ \sqrt{2}}{C_T^{3/2} \big/ \sqrt{2} + \frac{\sigma C_{d0}}{8}}$$

$$\text{If } \mathrm{C_T} = \sigma \overline{C}_l / 6$$

$$\text{FM is maximized if } \mathrm{C_{d0}} / \overline{C}_l^{3/2}$$

$$\text{is minimized.}$$