Hover Performance Prediction Methods

I. Momentum Theory

Also called

Actuator Disk Model

Background

- Developed for marine propellers by Rankine (1865), Froude (1885).
- Extended to include swirl in the slipstream by Betz (1920)
- This theory can predict performance in hover, and vertical climb.
- We will look at the general case of vertical climb, and extract hover as a special situation with zero climb velocity.

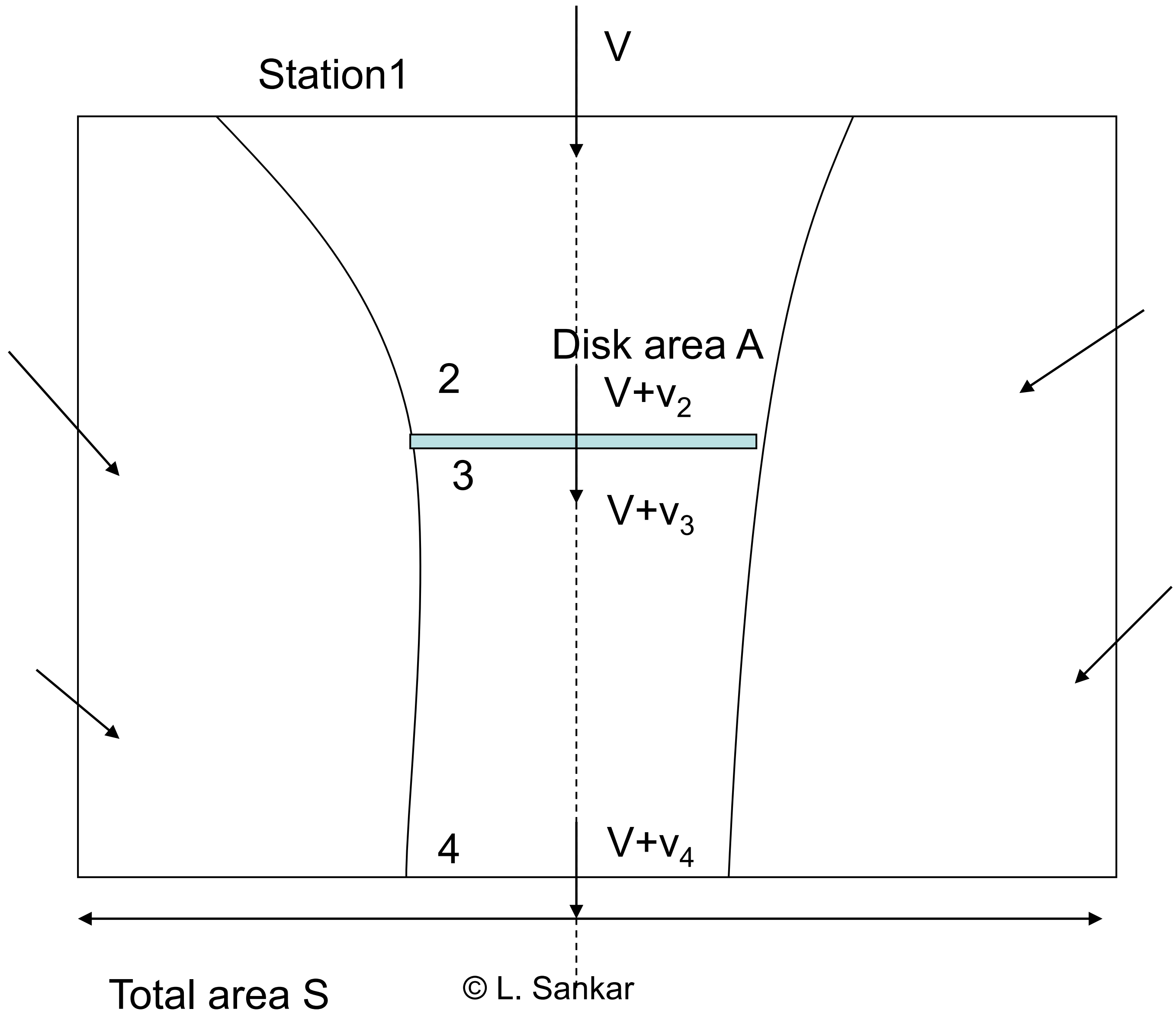
Assumptions

- Momentum theory concerns itself with the global balance of mass, momentum, and energy.
- It does not concern itself with details of the flow around the blades.
- It gives a good representation of what is happening far away from the rotor.
- This theory makes a number of simplifying assumptions.

Assumptions (Continued)

- Rotor is modeled as an actuator disk which adds momentum and energy to the flow.
- Flow is incompressible.
- Flow is steady, inviscid, irrotational.
- Flow is one-dimensional, and uniform through the rotor disk, and in the far wake.
- There is no swirl in the wake.

Control Volume is a Cylinder



Conservation of Mass

Inflow through the top $= \rho \mathrm{VS}$

Inflow through the side $= \dot{\mathrm{m}}_1$

Outflow through the bottom $= \rho \mathrm{V}\left(\mathrm{S} - \mathrm{A}_4\right) + \rho (V + v_4) A_4$

Total inflow must equal total outflow in steady flow.

$$\dot{\mathrm{m}}_1 = \rho \mathrm{v}_4 A_4$$

Conservation of Mass through the Rotor Disk

Flow through the rotor disk =

$$\dot{m} = \rho A(V + \mathrm{v}_2) = \rho A(V + \mathrm{v}_3)$$
$$= \rho A_4(V + \mathrm{v}_4)$$

Thus $\mathrm{v}_2 = \mathrm{v}_3 = \mathrm{v}$

There is no velocity jump across the rotor disk

The quantity v is called induced velocity at the rotor disk

Global Conservation of Momentum

$$\text{Momentum inflow through top} = \rho V^2 S$$

$$\text{Momentum inflow through the side} = \dot{m}_1 V$$

$$= \rho A_4 v_4 V$$

Momentum outflow through bottom =

$$\rho(S - A_4)V^2 + \rho(V + v_4)^2 A_4$$

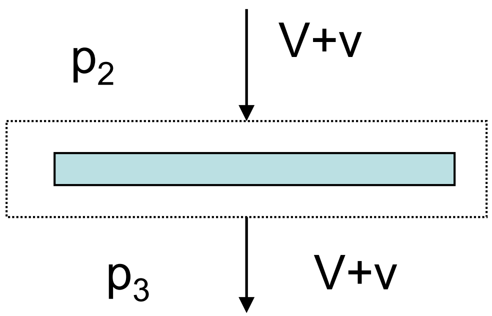
Pressure is atmospheric on all the far field boundaries.

Thrust, T = Momentum rate out - Momentum Rate in

$$T = \rho A_4 (V + v_4) v_4 = \dot{m} v_4$$

Mass flow rate through the rotor disk times
Excess velocity between stations 1 and 4

Conservation of Momentum at the Rotor Disk

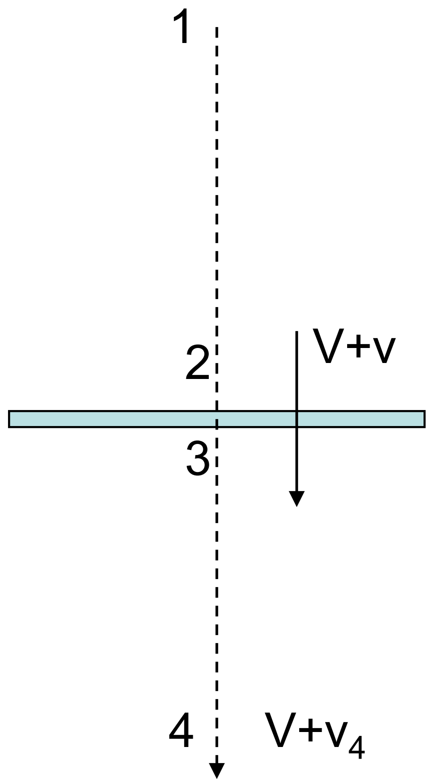


Due to conservation of mass across the Rotor disk, there is no velocity jump.

Momentum inflow rate = Momentum outflow rate

Thus, Thrust $T = A(p_3-p_2)$

Conservation of Energy



Consider a particle that traverses from Station 1 to station 4

We can apply Bernoulli equation between Stations 1 and 2, and between stations 3 and 4.
Recall assumptions that the flow is steady, irrotational, inviscid.

$$p_2 + \frac{1}{2}\rho(V + \mathrm{v})^2 = p_\infty + \frac{1}{2}\rho V^2$$

$$p_3 + \frac{1}{2}\rho(V + \mathrm{v})^2 = p_\infty + \frac{1}{2}\rho(V + \mathrm{v}_4)^2$$

$$p_3 - p_2 = \rho\left(V + \frac{\mathrm{v}_4}{2}\right)\mathrm{v}_4$$

From the previous slide #38,

$$p_3 - p_2 = \rho\left(V + \frac{\mathrm{v}_4}{2}\right)\mathrm{v}_4$$

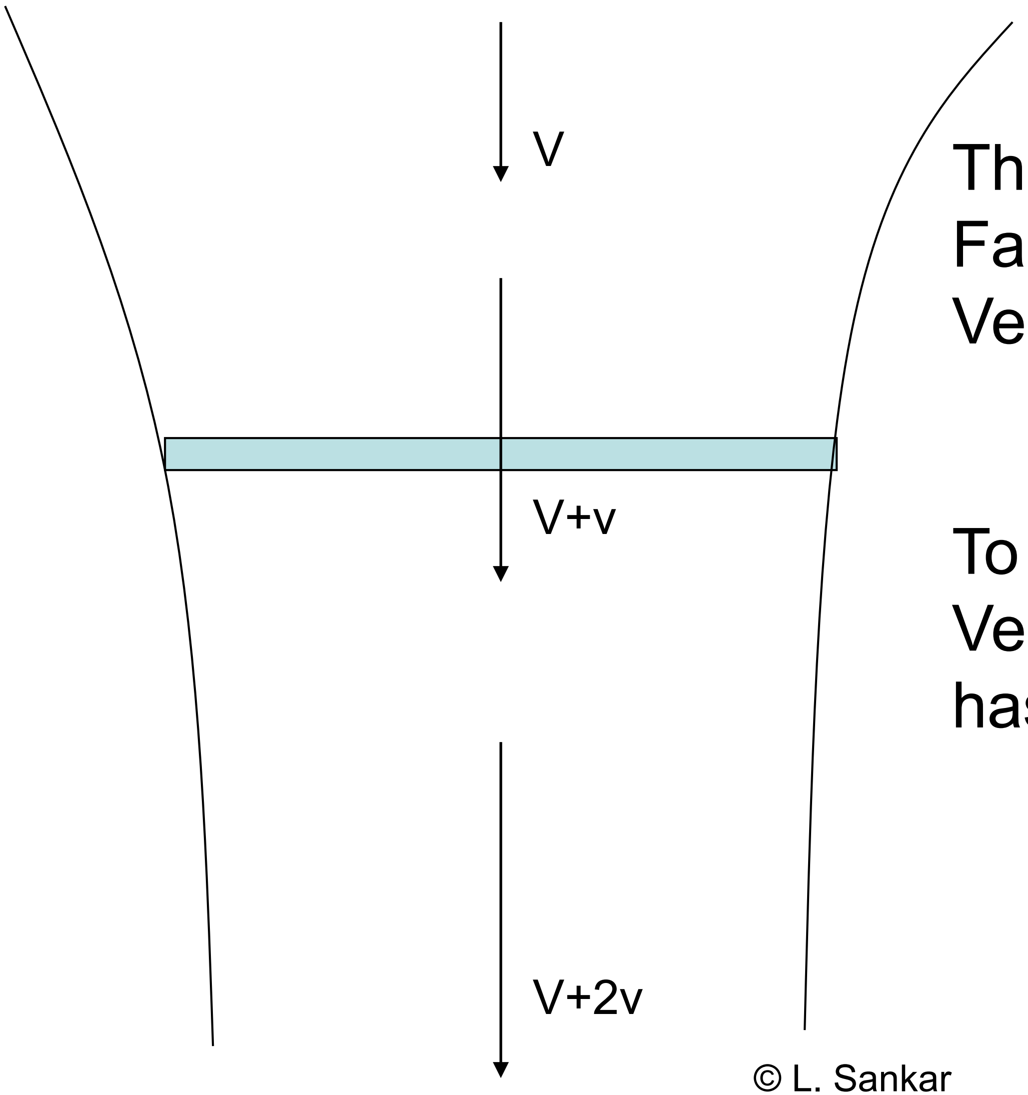
$$T = A\left(p_3 - p_2\right) = \rho A\left(V + \frac{\mathrm{v}_4}{2}\right)\mathrm{v}_4$$

From an earlier slide # 36, Thrust equals mass flow rate through the rotor disk times excess velocity between stations 1 and 4

$$T = \rho A\left(V + \mathrm{v}\right)\mathrm{v}_4$$

Thus, $\mathrm{v} = \mathrm{v}_4/2$

Induced Velocities



The excess velocity in the Far wake is twice the induced Velocity at the rotor disk.

To accommodate this excess Velocity, the stream tube has to contract.

Induced Velocity at the Rotor Disk

Now we can compute the induced velocity at the rotor disk in terms of thrust T.

T = Mass flow rate through the rotor disk * (Excess velocity between 1 and 4).

$T = 2\, \rho\, A\, (V+v)\, v$

$$v = -\frac{V}{2} \pm \sqrt{\left(\frac{V}{2}\right)^2 + \frac{T}{2\rho A}}$$

There are two solutions. The – sign Corresponds to a wind turbine, where energy Is removed from the flow. v is negative.

The + sign corresponds to a rotor or Propeller where energy is added to the flow. In this case, v is positive.

Induced velocity at the rotor disk

$$\mathrm{v} = -\frac{\mathrm{V}}{2} + \sqrt{\left(\frac{V}{2}\right)^2 + \frac{T}{2\rho A}}$$

In Hover, climb velocity V = 0

$$\mathrm{v} = \sqrt{\frac{T}{2\rho A}}$$

Ideal Power Consumed by the Rotor

$$P = \text{Energy flow out - Energy flow in}$$

$$= \frac{1}{2}\dot{m}\left(\mathrm{V} + 2\mathrm{v}\right)^2 - \frac{1}{2}\dot{m}\mathrm{V}^2$$

$$= 2\dot{m}\mathrm{v}\left(V + \mathrm{v}\right)$$

$$= T\left(V + \mathrm{v}\right)$$

$$= T\left[\frac{V}{2} + \sqrt{\left(\frac{V}{2}\right)^2 + \frac{T}{2\rho A}}\right]$$

In hover, ideal power $= T\sqrt{\dfrac{T}{2\rho A}}$

Summary

- According to momentum theory, the downwash in the far wake is twice the induced velocity at the rotor disk.
- Momentum theory gives an expression for induced velocity at the rotor disk.
- It also gives an expression for ideal power consumed by a rotor of specified dimensions.
- Actual power will be higher, because momentum theory neglected many sources of losses- viscous effects, compressibility (shocks), tip losses, swirl, non-uniform flows, etc.

Figure of Merit

- Figure of merit is defined as the ratio of ideal power for a rotor in hover obtained from momentum theory and the actual power consumed by the rotor.
- For most rotors, it is between 0.7 and 0.8.

$$FM = \frac{\text{Ideal Power in Hover}}{\text{Actual Power in Hover}}$$

$$= \frac{T\text{v}}{\text{P}} = \frac{C_T \sqrt{\frac{C_T}{2}}}{C_P}$$

Some Observations on Figure of Merit

- Because a helicopter spends considerable portions of time in hover, designers attempt to optimize the rotor for hover (FM~0.8).
- We will discuss how to do this later.
- A rotor with a lower figure of merit (FM~0.6) is not necessarily a bad rotor.
- It has simply been optimized for other conditions (e.g. high speed forward flight).

Example #1

- A tilt-rotor aircraft has a gross weight of 60,500 lb. (27500 kg).
- The rotor diameter is 38 feet (11.58 m).
- Assume FM=0.75, Transmission losses=5%
- Compute power needed to hover at sea level on a hot day.

Example #1 (Continued)

$$\text{Disk Area} = \text{A} = \pi(19)^2$$

$$A = 1134.12 \text{ square feet}$$

$$\text{Density} = 0.00238 \text{ slugs/cubic feet}$$

$$\text{There are two rotors. T} = 30250 \text{ lbf}$$

$$\text{Induced velocity, v} = \sqrt{\frac{\text{T}}{2\rho\text{A}}}$$

$$\text{v} = 74.86 \text{ ft/sec}$$

$$\text{Downwash in the far wake} \cong 150 \text{ ft/sec!}$$

$$\text{Ideal Power} = \text{Tv} = 30250 \text{ x } 74.86 \text{ lb ft/sec}$$

$$\text{Ideal Power} = 4117 \text{ HP}$$

$$\text{Actual Power} = \text{ideal Power/FM} = 4117/0.75$$

$$\text{Actual power} = 5490 \text{ HP}$$

$$\text{For the two rotors, total actual power} = 10980 \text{ HP}$$

$$\text{There is 5\% transmission loss}$$

$$\text{Power supplied by the engine to the shaft} = 10980 * 1.05 = 11528 \text{ HP}$$

Alternate scenarios

- What happens on a hot day, and/or high altitude?
  - Induced velocity is higher.
  - Power consumption is higher
- What happens if the rotor disk area A is smaller?
  - Induced velocity and power are higher.
- There are practical limits to how large A can be.

Disk Loading

- The ratio T/A is called disk loading.
- The higher the disk loading, the higher the induced velocity, and the higher the power.
- For helicopters, disk loading is between 5 and 10 lb/ft^2 (24 to 48 kg/m^2).
- Tilt-rotor vehicles tend to have a disk loading of 20 to 40 lbf/ft^2. They are less efficient in hover.
- VTOL aircraft have very small fans, and have very high disk loading (500 lb/ft^2).

Power Loading

- The ratio of thrust to power T/P is called the Power Loading.
- Pure helicopters have a power loading between 6 to 10 lb/HP.
- Tilt-rotors have lower power loading – 2 to 6 lb/HP.
- VTOL vehicles have the lowest power loading – less than 2 lb/HP.

Non-Dimensional Forms

Thrust, Torque, and Power are usually expressed in non - dimensional form.

$$C_T = \text{Thrust Coefficient} = \frac{T}{\rho A(\Omega R)^2}$$

$$C_P = \text{Power Coefficient} = \frac{P}{\rho A(\Omega R)^3}$$

$$C_Q = \text{Torque Coefficient} = \frac{Q}{\rho AR(\Omega R)^2}$$

In hover, Power = Angualr velocity x Torque

$$P = \Omega Q$$

$$C_P = C_Q$$

Non-dimensional forms..

$$\text{Induced inflow} = \lambda_i = \frac{\mathrm{v}}{\Omega \mathrm{R}} = \frac{1}{\Omega \mathrm{R}} \sqrt{\frac{\mathrm{T}}{2\rho \mathrm{A}}} = \sqrt{\frac{\mathrm{C_T}}{2}}$$

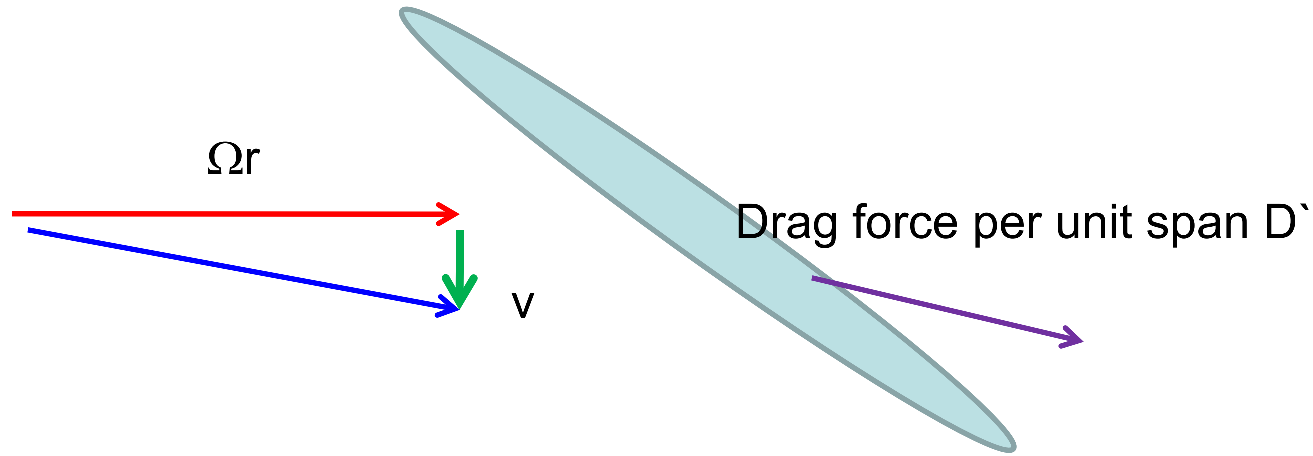
$$FM = \frac{\text{Ideal Power in Hover}}{\text{Actual Power in Hover}}$$

$$= \frac{T\mathrm{v}}{\mathrm{P}} = \frac{C_T \sqrt{\frac{C_T}{2}}}{C_P}$$

Accounting for Viscous Losses

- Helicopter blades are made of airfoil sections.
- The blades experience a viscous resistive force, called viscous drag or “profile drag”.
- We will use the symbol D` to represent the drag per unit span, e.g. lbf/per foot of span or Newtons per meter of the span.

Accounting for viscous losses

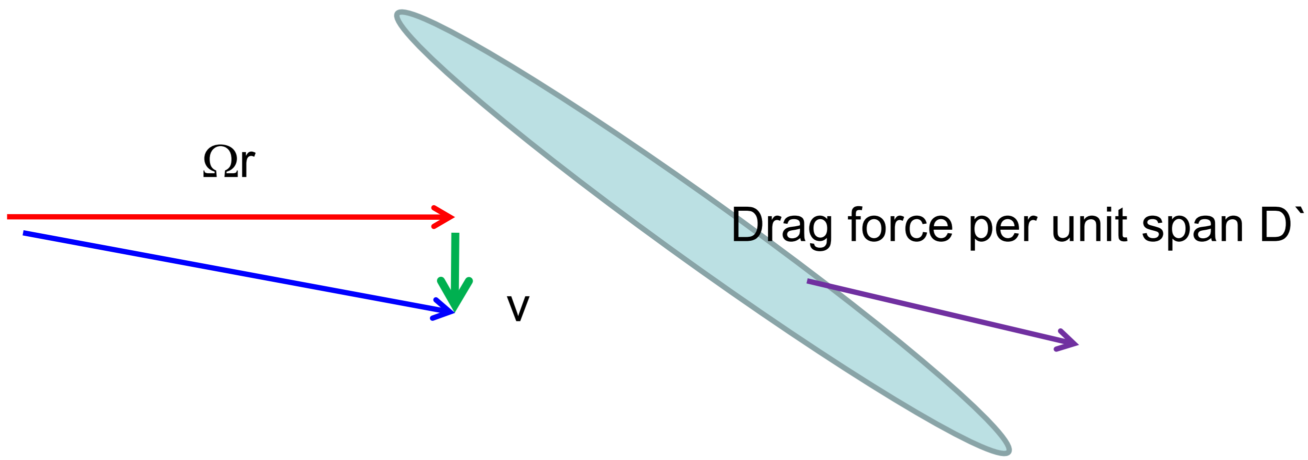


Power consumed by this drag force per unit span = D` (Ωr)

We are neglecting the induced velocity v.

Induced velocity is small, of order of 10 to 70 feet/sec whereas (Ωr) is
Of order of hundreds of feet/sec.

Accounting for Viscous losses



From classical aerodynamics, drag coefficient is given by

$$C_d = \frac{D'}{\frac{1}{2}\rho(Velocity\_squared)c}$$

$$= \frac{D'}{\frac{1}{2}\rho\left((\Omega r)^2 + v^2\right)c} \approx \frac{D'}{\frac{1}{2}\rho\left((\Omega r)^2\right)c}$$

where c is chord of the airfoil.

Power consumed by Viscous Drag

- Every foot of span (or meter of span) will consume power equal to D` times (Ωr)
- If we integrate to D` times (Ωr) from root to tip, we can get total power per blade.
- If we multiply the result by b, where b is the symbol for the total number of blades, we get the total power due to viscous drag.
- We call this “Profile Power”

Power Consumed by Viscous Drag

$$P_{\mathrm{Pr}ofile} = b \int_{root}^{tip} D'(\Omega r) dr$$

$$= b \int_0^R \frac{1}{2} \rho c C_d (\Omega r)^2 \Omega r dr$$

$$\cong \frac{bc}{8} \rho C_d (\Omega R)^3 R$$

We have assumed rectangular blades, and a constant value of C_d

Profile Power

$$P_{\mathrm{Profile}} \cong \frac{bc}{8}\rho C_d (\Omega R)^3 R$$

$$C_{P,\mathrm{Profile}} = C_{P,0} = \frac{P_{\mathrm{Profile}}}{\rho A (\Omega R)^3}$$

$$= \frac{bcR}{\pi R^2}\frac{C_d}{8}$$

The first term on the right side is blade area (bcR)
Divided by disk area.

We call this ratio solidity, σ.

Helicopters typically have a solidity between 0.05 and 0.1

That is, less than 10% of the disk is blades, rest is air.

Total Power

- If we add the ideal power we looked earlier, to the profile power, we get, in hover,

$$C_{P,induced} = C_T \sqrt{\frac{C_T}{2}}$$

$$C_{P,\mathrm{Pr}ofile} = \sigma \frac{C_d}{8}$$

$$C_P = C_T \sqrt{\frac{C_T}{2}} + \sigma \frac{C_d}{8}$$

Total Power

- We may have other sources of losses (e.g. swirl) that we alluded to earlier.
- To account for these, the induced power is multiplied by a factor k, approximately 1.15.

$$C_P = \kappa C_T \sqrt{\frac{C_T}{2}} + \sigma \frac{C_d}{8}$$